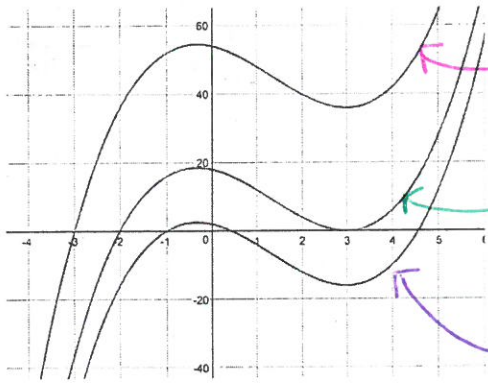
Notes 15-2 Polynomial Functions and their Graphs and Zeros

Let's compare these three polynomial functions:

$f(x) = x^3 - 4x^2 - 3x + 2$ $\quad g(x) = x^3 - 4x^2 - 3x + 18$ $\quad h(x) = x^3 - 4x^2 - 3x + 54$



$h(x) \rightarrow$ has 1 real root - neg & 2 imag. roots

$g(x) \rightarrow$ has 3 real roots (1 neg, 1 pos d.r.)
double root (d.r.) @ 3

$f(x) \rightarrow$ has 3 real roots (1 neg, 2 pos)
(crosses x-axis 3 times)

A polynomial function of degree n can have up to n increasing and decreasing branches resulting in up to n zeros and up to $n - 1$ extreme points. (max or min points)

Recall: • degree of polynomial tells # of roots

• leading coefficient describes end behavior
- if neg, y goes to $-\infty$
- if pos, y goes to $+\infty$

} see above -- all l.c. are + all y's go to $+\infty$

• # of branches = # roots = degree of polynomial

• even degree - starts & ends same direction
odd degree - starts & ends in opp. direction

• can't be an odd # of imag. roots $\rightarrow$ they come in pairs

** Synthetic Div. review

$h(x) = x^3 - 4x^2 - 3x + 54$ has 1 real root $\Rightarrow x = -3$

* to find other factor and other roots:

$$\begin{array}{r|rrrr} -3 & 1 & -4 & -3 & 54 \\ & \downarrow & -3 & 21 & -54 \\ \hline & 1 & -7 & 18 & \boxed{0} \end{array}$$

$x^2 - 7x + 18$

remainder 0 confirms it's a root

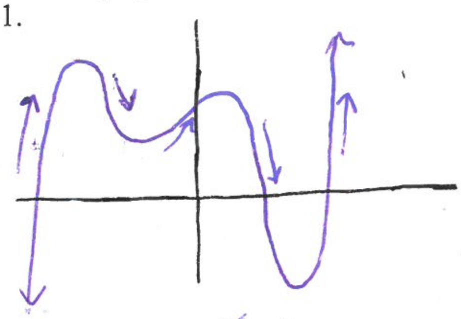
* put quad in calc for imag. roots.

$\boxed{\text{Roots: } -3, 3.5 \pm 2.4i}$

$\boxed{\text{Factors: } (x+3)(x^2-7x+18)}$

For the polynomial functions below, find:

1.



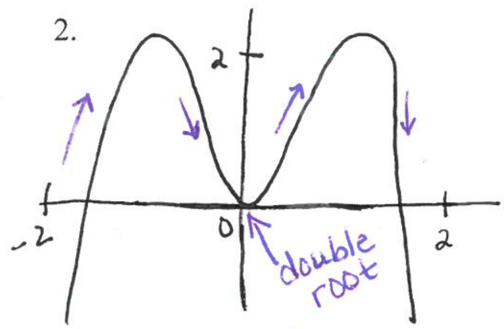
Degree 5

# of real zeros 3

# of complex zeros 2

leading coefficient sign +

2.



Degree 4

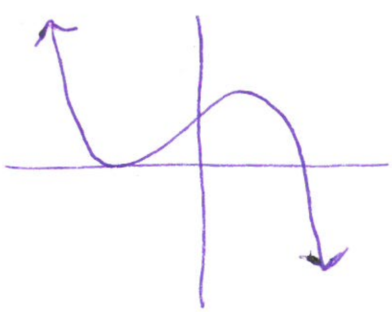
# of real zeros 4

# of complex zeros 0

leading coefficient sign −

Sketch the graph of the polynomial function described or explain why no function can exist.

3. cubic function with a negative double zero and a positive zero, and a negative leading coefficient



4. quartic function with two distinct positive zeros, two distinct negative zeros, and a negative y-intercept