

Review: 5.2 and 5.3

Odd/even, cofunctions, composite argument properties

Part D: Prove that each of the following is an identity. Plug the left side in the ~~sum/difference~~ ^{composite argument} formulas and simplify.

19. $\sin(\theta + 60^\circ) - \cos(\theta + 30^\circ) = \sin\theta$

$$\begin{aligned} & [\sin\theta \cdot \cos 60 + \cos\theta \cdot \sin 60] - [\cos\theta \cdot \cos 30 - \sin\theta \cdot \sin 30] \\ & \frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta - [\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta] \\ & \frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta - \frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta \\ & \sin\theta \end{aligned}$$

20. $\sin(\theta + 30^\circ) + \cos(\theta + 60^\circ) = \cos\theta$

$$\begin{aligned} & [\sin\theta \cdot \cos 30 + \cos\theta \cdot \sin 30] + [\cos\theta \cdot \cos 60 - \sin\theta \cdot \sin 60] \\ & \frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta + \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta \\ & \cos\theta \end{aligned}$$

Part E: Simplify using composite argument properties.

21. $\sin\left(\frac{3x}{7}\right)\cos\left(\frac{4x}{7}\right) + \cos\left(\frac{3x}{7}\right)\sin\left(\frac{4x}{7}\right)$

$$\begin{aligned} & \sin\left(\frac{3x}{7} + \frac{4x}{7}\right) \\ & \sin x \end{aligned}$$

22. $\cos 65^\circ \cos 20^\circ + \sin 65^\circ \sin 20^\circ$

$$\begin{aligned} & \cos(65 - 20) \\ & \cos 45 \\ & \frac{\sqrt{2}}{2} \end{aligned}$$

Part F: Using only angles from special right triangles, rewrite the following as a sum or difference of two angles. Then find the exact value using the ~~sum or difference~~ ^{composite argument}.

23. $\cos 75^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$
 $\cos(45 + 30)$

$$\begin{aligned} & \cos 45 \cdot \cos 30 - \sin 45 \cdot \sin 30 \\ & \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \\ & \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \end{aligned}$$

24. $\sin 15^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$
 $\sin(45 - 30)$

$$\begin{aligned} & \sin 45 \cdot \cos 30 - \cos 45 \cdot \sin 30 \\ & \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \\ & \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \end{aligned}$$

25. $\tan 105^\circ = \frac{1 + \sqrt{3}}{1 - \sqrt{3}}$
 $\tan(60 + 45)$

$$\begin{aligned} & \frac{\tan 60 + \tan 45}{1 - \tan 60 \cdot \tan 45} \\ & \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \end{aligned}$$

26. $\cos 285^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$
 $\cos(150 + 135)$

$$\begin{aligned} & \cos 150 \cdot \cos 135 - \sin 150 \cdot \sin 135 \\ & -\frac{\sqrt{3}}{2} \cdot -\frac{\sqrt{2}}{2} - \left[\frac{1}{2} \cdot \frac{\sqrt{2}}{2}\right] \\ & \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \end{aligned}$$

27. $\sin \frac{5\pi}{12} = \frac{\sqrt{6} + \sqrt{2}}{4}$

$$\sin\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right)$$

$$\sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

$$\begin{aligned} & \sin \frac{\pi}{4} \cdot \cos \frac{\pi}{6} + \cos \frac{\pi}{4} \cdot \sin \frac{\pi}{6} \\ & \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \end{aligned}$$

28. $\tan \frac{11\pi}{12} = \frac{-3 + \sqrt{3}}{3 + \sqrt{3}}$

$$\tan\left(\frac{9\pi}{12} + \frac{2\pi}{12}\right) \quad \tan\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$$

$$\frac{\tan \frac{3\pi}{4} + \tan \frac{\pi}{6}}{1 - \tan \frac{3\pi}{4} \cdot \tan \frac{\pi}{6}}$$

$$\frac{-1 + \frac{\sqrt{3}}{3}}{1 + 1\left(\frac{\sqrt{3}}{3}\right)}$$

$$\frac{-1 + \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$$

$$\frac{-\frac{3}{3} + \frac{\sqrt{3}}{3}}{\frac{3}{3} + \frac{\sqrt{3}}{3}}$$

$$\frac{-3 + \sqrt{3}}{3 + \sqrt{3}}$$

$$\frac{-3 + \sqrt{3}}{3 + \sqrt{3}}$$

Review: 5.2 and 5.3
Odd/even, cofunctions, composite argument properties

Part A: Use the odd / even properties to rewrite each with positive arguments.

Recall: $\sin(-x) = -\sin x$; $\cos(-x) = \cos x$; $\tan(-x) = -\tan x$

- | | | | |
|--------------------------|--------------------------------|---|---|
| 1. $\sin(-13)^\circ =$ | <u>$-\sin(13)$</u> | 2. $\cos\left(-\frac{\pi}{6}\right) =$ | <u>$\cos\left(\frac{\pi}{6}\right)$</u> |
| 3. $\tan(-135^\circ) =$ | <u>$-\tan(135)$</u> | 4. $\sec(-\pi) =$ | <u>$\sec(\pi)$</u> |
| 5. $-\sec(-73^\circ) =$ | <u>$-\sec(73)$</u> | 6. $-\tan\left(-\frac{\pi}{5}\right) =$ | <u>$\tan\left(\frac{\pi}{5}\right)$</u> |
| 7. $-\sin(-305^\circ) =$ | <u>$\sin(305)$</u> | 8. $-\cos\left(-\frac{\pi}{3}\right) =$ | <u>$-\cos\left(\frac{\pi}{3}\right)$</u> |

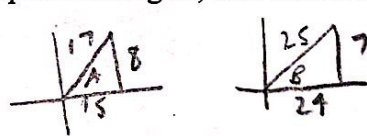
Part B: Use the cofunction properties to rewrite the following: (pos. arguments)

- | | | | |
|----------------------------|--|-----------------------------|---|
| 9. $\sin 25^\circ =$ | <u>$\cos(65)$</u> | 10. $\sec 12^\circ =$ | <u>$\csc(78)$</u> |
| 11. $\tan 88^\circ =$ | <u>$\cot(2)$</u> | 12. $\csc 46^\circ =$ | <u>$\sec(44)$</u> |
| 13. $\cot 13^\circ =$ | <u>$\tan(77)$</u> | 14. $\cos 90^\circ =$ | <u>$\sin(0)$</u> |
| 15. $\sin \frac{\pi}{4} =$ | <u>$\cos\left(\frac{\pi}{4}\right)$</u> | 16. $\cos \frac{2\pi}{3} =$ | <u>$\sin\left(-\frac{\pi}{6}\right)$</u> |

$\frac{\pi}{2} - \frac{2\pi}{3}$
 $\frac{3\pi}{6} - \frac{4\pi}{6}$

Part C: If α and β are the measures of two first quadrant angles, find the exact value of each function.

17. If $\sin A = \frac{8}{17}$ and $\tan B = \frac{7}{24}$, find $\cos(A - B)$



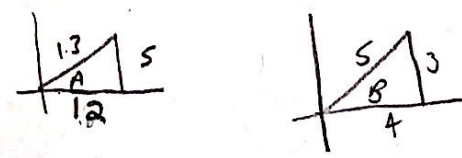
$$\cos A \cdot \cos B + \sin A \cdot \sin B$$

$$\frac{15}{17} \cdot \frac{24}{25} + \frac{8}{17} \cdot \frac{7}{25}$$

$$\frac{360}{425} + \frac{56}{425}$$

$\frac{416}{425}$

18. If $\csc A = \frac{13}{5}$ and $\tan B = \frac{3}{4}$, find $\sin(A + B)$



$$\sin A \cdot \cos B + \cos A \cdot \sin B$$

$$\left(\frac{5}{13}\right)\left(\frac{4}{5}\right) + \left(\frac{12}{13}\right)\left(\frac{3}{5}\right)$$

$$\frac{20}{65} + \frac{36}{65}$$

$\frac{56}{65}$