Section 7-4 Logarithms

We know that $2^2 = 4$ and $2^3 = 8$. However, for what value of x does $2^x = 6$?

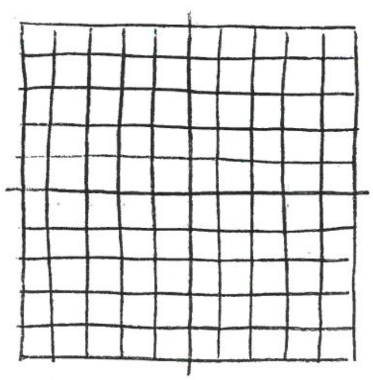
To find the exact value, mathematicians invented ***logarithms.***

> Let b and x be positive numbers, $b \neq 1$. The logarithm of x with base b is
>
> $$\log_b x = y \quad \text{if and only if} \quad b^y = x$$
>
> It is read as "log base b of x".

Logarithms and Exponential Functions are inverses of each other

$y = 2^x$ its inverse $x = 2^y$ or $\log_2 x = y$

x	y
-2	¼
-1	½
0	1
1	2
2	4



x	y
¼	-2
½	-1
1	0
2	1
4	2

Rewrite as an exponential function.

1. $\log_3 9 = 2$

$3^2 = 9$

2. $\log_5 \frac{1}{25} = -2$

$5^{-2} = \frac{1}{25}$

Rewrite an a logarithm.

3. $4^3 = 64$

$\log_4 64 = 3$

4. $10^4 = 10{,}000$

$\log_{10} 10{,}000 = 4$

The log with base 10 is called the **common logarithm**. It is written $\log_{10} x$ or $\log x$.
The log with base e =2.7182...is called the **natural logarithm.**
It can be written $\log_e x$ but is more often referred to as ln

Let b, u, and v be positive numbers such that $b \neq 1$.

Product Property $\log_b uv = \log_b u + \log_b v$

Example: $\log_5 21 = \log_5 3 + \log_5 7$

Quotient Property $\log_b \frac{u}{v} = \log_b u - \log_b v$

Example: $\log_5 \frac{3}{7} = \log_5 3 - \log_5 7$

Power Property $\log_b u^n = n \log_b u$

Example: $\log_5 49 = \log_5 7^2 = 2\log_5 7$

Demonstrate numerically the property of logarithms.

5. $\ln(7 \cdot 8) = \ln 7 + \ln 8$

$\ln 56 = 1.9459 + 2.0794$

$4.025 = 4.025$

Fill in the blank.

6. $\log 5 + \log 8 = \log$ 40

7. $\ln 4 - \ln 20 = \ln$ $\frac{4}{20}$ $\ln \frac{1}{5}$

8. $\log 49 =$ 2 $\log 7$

$\log 7^2$

9. $\log 100 =$ 2

$10^x = 100$

CHANGE-OF-BASE formula

$\log_c u = \dfrac{\log u}{\log c}$ or $\log_c u = \dfrac{\ln u}{\ln c}$

$\log_3 7 = \dfrac{\log 7}{\log 3}$

10. $\log_2 6$

$\dfrac{\log 6}{\log 2}$

11. $\log_5 129$

$\dfrac{\log 129}{\log 5}$

12. $\log_{1/2} 7$

$\dfrac{\log 7}{\log \frac{1}{2}}$

Solve.

13. $4^x = 15$

$x \cdot \log 4 = \log 15$

$x = \dfrac{\log 15}{\log 4}$ $\boxed{x = 1.95}$

14. $3^{4x} = 27^{x+1}$

$(4x) \log 3 = (x+1) \log 27$

$1.9085x = 1.4314x + 1.4314$

$.4771x = 1.4314$

$x = 3.00020$ $\boxed{x = 3}$

15. $\log_5(x+6) + \log_5(x+2) = 1$

$\log_5 (x+6)(x+2) = 1$

$5^1 = (x+6)(x+2)$

$5 = x^2 + 6x + 2x + 12$

$0 = x^2 + 8x + 7$

$0 = (x+7)(x+1)$ $\cancel{x = -7}$ $\boxed{x = -1}$

16. $\log_2(2x-1) - \log_2(x+2) = -1$

$\log_2 \dfrac{2x-1}{x+2} = -1$

$2^{-1} = \dfrac{2x-1}{x+2}$

$\dfrac{1}{2} = \dfrac{2x-1}{x+2}$

$x + 2 = 4x - 2$

$4 = 3x$

$\boxed{\frac{4}{3} = x}$

17. $e^{2x} - 3e^x + 2 = 0$

quadratic

$(e^x)^2 - 3(e^x) + 2 = 0$

$e^x = 2$ $e^x = 1$

$\cancel{\ln e}^x = \ln 2$ $\cancel{\ln e}^x = \ln 1$

$\boxed{x = 0.6931 \quad x = 0}$