Section 7-6 Logistic Functions

Logistic Functions are used when growth levels off (approaches an asymptote).

$$y = \frac{c}{1 + ab^{-x}}$$

where a, b, and c are constants and the domain is all real numbers.

* c is the 'cap' or the limit

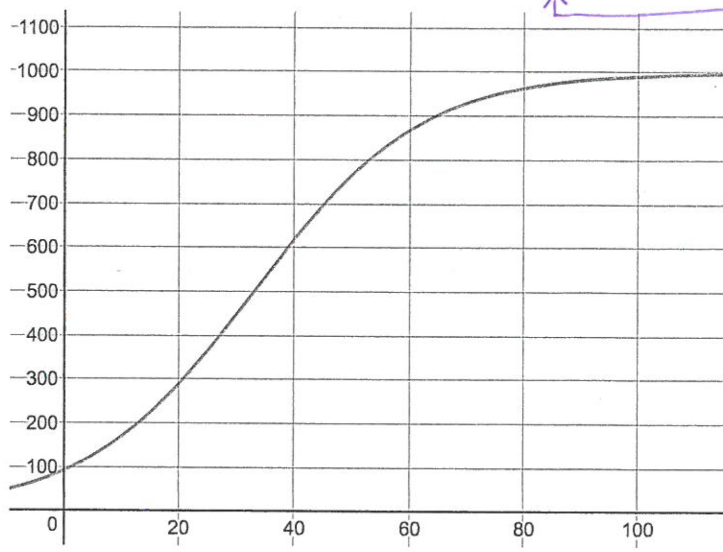
** → this is NOT an unknown

Example:

Suppose that the population of a new subdivision is growing rapidly. Look at the table of monthly population in # of houses in the sub division. Suppose that there are only 1000 lots in the subdivision.

'c'

x (months)	y (houses)
2	103
4	117
6	132
8	148
10	167



a. Use (2, 103) and (10, 167) to find the particular equation of the logistic function.

$y = \frac{c}{1+ab^{-x}}$

* 'c' is limit or 'cap'

$103 = \frac{1000}{1+ab^{-2}}$

$167 = \frac{1000}{1+ab^{-10}}$

$1+ab^{-2} = \frac{1000}{103}$

$1+ab^{-10} = \frac{1000}{167}$

$ab^{-2} = \left(\frac{1000}{103}\right) - 1$

$ab^{-10} = \left(\frac{1000}{167}\right) - 1$

* a+b are mult, so div.

subt. exp. → $-2-(-10)$

$b^{8} = \frac{\left(\frac{1000}{103}\right) - 1}{\left(\frac{1000}{167}\right) - 1}$

* take the 8th root of both sides

$b \approx 1.0721$

* plug back in to find a

$a(1.0721)^{-2} = \left(\frac{1000}{103}\right) - 1$

$a \approx 10.01$

$$\boxed{y = \frac{1000}{1+10.01(1.07)^{-x}}}$$

b. Use the logistic function to predict the number of houses that will be occupied in two years. What process do you use, extrapolation or interpolation?

'y'

$X = \#$ mos

2 yrs = 24 mos

$$y = \frac{1000}{1 + 10.01(1.07)^{-24}}$$

$$\boxed{y = 336.31 \text{ houses}}$$

*extrapolation because x value of 24 is outside the given set of data

The **point of inflection** is halfway between the x-axis and the asymptote. Remember the asymptote is *c*.

c. Find the value of x at the point of inflection. What is the real-world meaning of this point?

pt. of inflection halfway b/w $y = 0$ & $y = 1000$, so @ $y = 500$

$$500 = \frac{1000}{1 + 10.01(1.07)^{-x}}$$

$$1 + 10.01(1.07)^{-x} = \frac{1000}{500}$$

$$10.01(1.07)^{-x} = 2 - 1$$

$$1.07^{-x} = \frac{1}{10.01}$$

$$-x \log 1.07 = \log\left(\frac{1}{10.01}\right)$$

$$-x = \frac{\log\left(\frac{1}{10.01}\right)}{\log(1.07)}$$

$x = 34.047$ $\quad x \approx 34$

$\boxed{\text{34 yrs. to be half full}}$