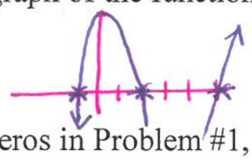
Chapter 15 Review **Name** Key

No calculator #1- 9

1. Write the zeros of the function $y = (x-5)(x^2-x-2)$

$x-5=0$ $\boxed{x=5}$ $x^2-x-2=0$ $(x-2)(x+1)=0$ $\boxed{x=2 \quad x=-1}$

2. Sketch the graph of the function in Problem #1. Show where the zeros are and what shape the graph is.



$*$ = zeros

3. Using the zeros in Problem #1, write the sum of the zeros, pair-wise products and the products of the zeros.

sum $= 5+2+(-1) = \boxed{6}$

product $= (5)(2)(-1) = \boxed{-10}$

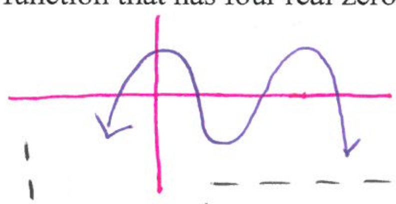
pwp $= (5)(2)+(5)(-1)+(2)(-1) = \boxed{3}$

$10 + (-5) + (-2)$

4. Write a cubic equation using the answers from #3 where the leading coefficient is 1. $\boxed{y=x^3-6x^2+3x+10}$

sum $= -\frac{b}{a} \Rightarrow 6$ $b=-6$ pwp $= \frac{c}{a} = 3$ $c=3$ prod. $= \frac{-d}{a} = -10$ $d=10$

5. Sketch the graph of a quartic function that has four real zeros and a positive y-intercept.



(just an example)

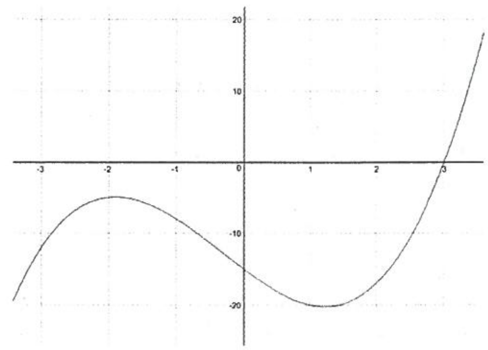
$y = x^3 + x^2 - 7x - 15$

6. Degree 3

#real zeros 1

#complex zeros 2

Sign of leading coefficient pos.



7. Using Problem #6: use synthetic substitution to show x = 3 is a zero.

3	1	1	-7	-15
	↓	3	12	15
	1	4	5	0

remainder = 0, so $x=3$ is a zero

8. Using the answer to #7, write the function as a linear times a quadratic.

$y = (x-3)(x^2+4x+5)$

9. Using the answer to #8, find the other two complex zeros of the cubic function.

$x^2+4x+5 \rightarrow$ quad. form.

$\boxed{\begin{matrix} 2+i \\ 2-i \end{matrix}}$

May use Calculator #10 – 24

10. Show that the data fits a cubic function by showing the 3rd differences are constant.

x	f(x)
2	19.4
3	40.1
4	74.2
5	123.5
6	189.8
7	274.9

20.7, 34.1, 49.3, 66.3, 85.1

13.4, 15.2, 17, 18.8

1.8, 1.8, 1.8

$$[A] = \begin{bmatrix} 8 & 4 & 2 & 1 & 19.4 \\ 27 & 9 & 3 & 1 & 40.1 \\ 64 & 16 & 4 & 1 & 74.2 \\ 125 & 25 & 5 & 1 & 123.5 \end{bmatrix}$$

rref [A]

11. Find the particular equation of the cubic function ALGEBRAICALLY.
12. Check your answer to #11 by running the cubic regression test on your calculator.

$f(x) = 0.3x^3 + 4x^2 - 5x + 11$

For #13-18: Stephanie drives through an intersection. At time t = 2sec she crosses the stripe at the beginning of the intersection. She slows down a bit, but does not stop and then speeds up again. Stephanie is good at math, and she figures that her displacement, d(t), in feet, from the first stripe is given by $d(t) = t^3 - 12t^2 + 54t - 68$

13. Use synthetic substitution to show that t = 2 is a zero of d(t).

$$\begin{array}{r|rrrr} 2 & 1 & -12 & 54 & -68 \\ & \downarrow & 2 & -20 & 68 \\ \hline & 1 & -10 & 34 & 0 \end{array}$$

no remainder, so 2 is a zero

14. Use the results from #13 to find the other two zeros of d(t).

$0 = x^2 - 10x + 34$ (quad form) $5+3i$ and $5-3i$

15. How do the zeros in #14 confirm the fact that Stephanie does not stop and go back across the stripe? they're imaginary which means graph doesn't cross x-axis again (1st stripe)

16. Find the derivative of d(t) using the short cut Power Rule.

$d'(t) = 3t^2 - 24t + 54$

17. Find $d'(3)$, which is Stephanie's instantaneous velocity at 3 seconds.

$d'(3) = 3(3)^2 - 24(3) + 54$ $\boxed{d'(3) = 9}$

18. Find the equation of the line tangent at t = 3. $y = mx + b$

$d(3) = y$ $d'(3) = m$ $x = 3$

$13 = 9(3) + b$

$b = -14$

$\boxed{y = 9x - 14}$

19. Using the derivative of $f(x) = x^3 - 2x^2 - 5x + 6$, find the x-coordinate of the extreme points of the graph f(x).

$f'(x) = 3x^2 - 4x - 5$ so $0 = 3x^2 - 4x - 5$ so $x = 2.12$, $x = -0.79$

(derivative = 0 @ extreme pts. because tangent line is horizontal at extreme pts. which means slope (derivative) = 0.

For #20-24: $f(x) = \frac{2x-6}{x^2+2x-15}$

20. What is the real zero of f(x)?

$0 = \frac{2x-6}{x^2+2x-15} \Rightarrow$ (mult by denom) $\Rightarrow 0 = 2x - 6 \Rightarrow 6 = 2x \Rightarrow \boxed{x = 3}$

21. Find any discontinuities of f(x). $\frac{2(x-3)}{(x+5)(x-3)}$ $\boxed{\text{disc. @ } x = -5 \text{ AND } x = 3}$

22. What kind of discontinuity are they?

removable @ x = 3 (because of common factor in num); non-removable @ x = −5

23. Find the limit as x approaches each discontinuity.

$\boxed{\lim_{x \to -5} f(x) = \infty}$

$\frac{2\cancel{(x-3)}}{(x+5)\cancel{(x-3)}} \Rightarrow \frac{2}{(3+5)} = \frac{2}{8}$

$\boxed{\lim_{x \to 3} f(x) = \frac{1}{4}}$

24. Resolve f(x) into partial fractions.

$\frac{2(x-3)}{(x-3)(x+5)} \Rightarrow \frac{2(-5-3)}{-5-3} = \frac{-16}{-8} = 2$

$\frac{2(3-3)}{3+5} = \frac{0}{8} = 0$

$\Rightarrow \boxed{\frac{2}{x+5} + \frac{0}{x-3}}$