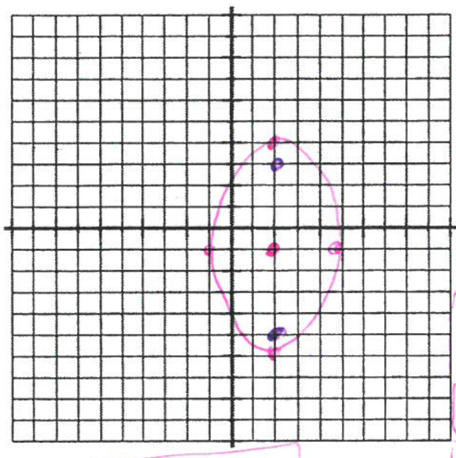
Review for Chapter 12

A) Graph. Label foci, vertices, asymptotes.
B) Then write parametric equations for each problem.
C) Find eccentricity, *e*.

1. $\frac{(x-2)^2}{9}+\frac{(y+1)^2}{25}=1$ $\quad \left(\frac{x-2}{3}\right)^2+\left(\frac{y+1}{5}\right)^2=1$



$a^2=b^2+c^2$

$25=9+c^2$

$c=4$

center $(2,-1)$

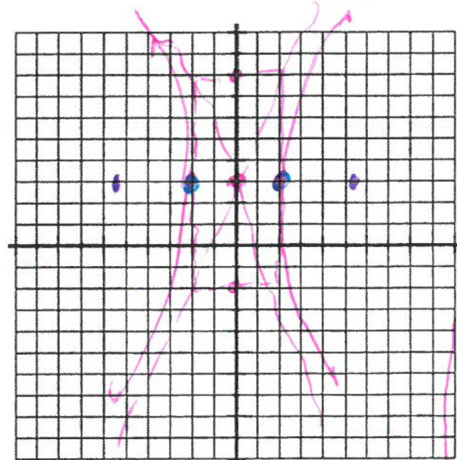
$\boxed{\text{foci } (2,-1\pm4)}$

$\boxed{(2,-5)\ (2,3)}$

$\boxed{e=\frac{4}{5}}$

$\boxed{\begin{array}{l} x=2+3\cos T \\ y=-1+5\sin T \end{array}}$

2. $\frac{x^2}{4}-\frac{(y-3)^2}{25}=1$ $\quad \left(\frac{x}{2}\right)^2-\left(\frac{y-3}{5}\right)^2=1$



$c^2=a^2+b^2$

$c^2=4+25$

$c=\sqrt{29}$

center $(0,3)$

foci $(0\pm\sqrt{29},3)$

vertices

$(-2,3)\ (2,3)$

$e=\frac{\sqrt{29}}{2}$

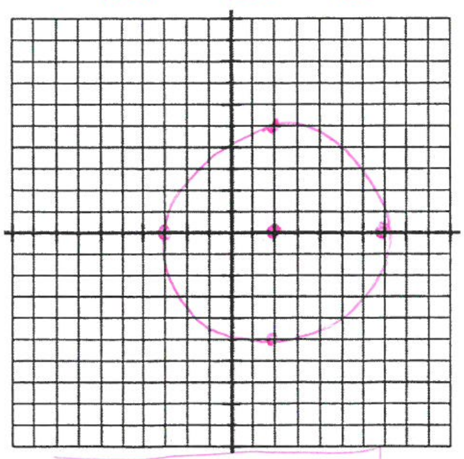
slopes of asymp. $=\pm\frac{5}{2}$

$\boxed{\begin{array}{l} x=2\sec T \\ y=3+5\tan T \end{array}}$

3. $5(x-2)^2+5y^2=125$

$\frac{5(x-2)^2}{125}+\frac{5y^2}{125}=\frac{125}{125}$ $\quad \frac{(x-2)^2}{25}+\frac{y^2}{25}=1$

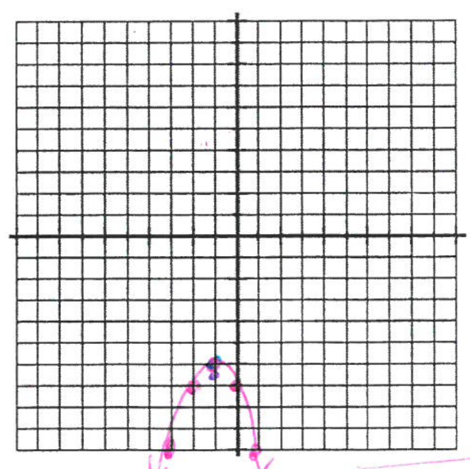
$\left(\frac{x-2}{5}\right)^2+\left(\frac{y}{5}\right)^2=1$



$\boxed{\begin{array}{l} x=2+5\cos T \\ y=0+5\sin T \end{array}}$

$\boxed{e=0}$

4. $(x+1)^2+y+6=0$



$y=-(x+1)^2-6$

$\boxed{\text{vertex } (-1,-6)}$

$p=\frac{1}{4(-1)}=-\frac{1}{4}$

$\boxed{\begin{array}{l} y=-T^2-6 \\ x=T-1 \end{array}}$

$\boxed{\begin{array}{l} \text{focus}=\left(-1,-6\tfrac{1}{4}\right) \\ e=1 \end{array}}$

Without graphing, tell whether the graph would be a circle, an ellipse, hyperbola, or parabola.

5. $5x^2 + 5y^2 = 125$ C

6. $x^2 - y^2 = 1$ H

7. $5(x-2)^2 + 5y = 11$ P

8. $2x^2 + 5y^2 = 100$ E

Transform each equation into standard form (normal looking).

9. $2x^2 + y^2 - 4x + 8y + 12 = 0$

$(2x^2-4x)+(y^2+8y)=-12$

$2(x^2-2x+1)+(y^2+8y+16)=-12 +1(2) +16$

$\frac{2(x-1)^2}{6}+\frac{(y+4)^2}{6}=\frac{6}{6}$

$\boxed{\frac{(x-1)^2}{3}+\frac{(y+4)^2}{6}=1}$

$\boxed{\left(\frac{x-1}{\sqrt{3}}\right)^2+\left(\frac{y+4}{\sqrt{6}}\right)^2=1}$

10. $x^2 + 6x - 2y - 5 = 0$

$(x^2+6x+9)=2y+5 +9$

$(x+3)^2=2y+14$

$\frac{(x+3)^2}{2}-\frac{14}{2}=\frac{2y}{2}$

$\boxed{y=\frac{1}{2}(x+3)^2-7}$

11. $3x^2 - 9y^2 + 12x + 18y - 6 = 0$

$(3x^2+12x)+(-9y^2+18y)=6$

$3(x^2+4x+4)-9(y^2-2y+1)^2=6+4(3) +1(-9)$

$\frac{3(x+2)^2}{9}-\frac{9(y-1)^2}{9}=\frac{9}{9}$

$\boxed{\frac{(x+2)^2}{3}-(y-1)^2=1}$

$\boxed{\left(\frac{x+2}{\sqrt{3}}\right)^2-\left(\frac{y-1}{1}\right)^2=1}$

12. $5x^2 + 5y^2 + 10x - 20y + 1 = 0$

$(5x^2+10x)+(5y^2-20y)=-1 +1(5)+4(5)$

$5(x^2+2x+1)+5(y^2-4y+4)=-1$

$5(x+1)^2+5(y-2)^2=24$

$\boxed{(x+1)^2+(y-2)^2=\frac{24}{5}}$ OR

$\boxed{\frac{(x+1)^2}{\frac{24}{5}}+\frac{(y-2)^2}{\frac{24}{5}}=1}$

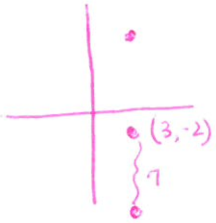
Sketch the quadric surface described. Not on test -- EC

13. paraboloid formed by rotating the graph $x = 4 - y^2$ that lies in the first quadrant about the x-axis

14. Ellipsoid formed by rotating the graph $4x^2 + y^2 = 16$ about the y-axis

Write the equation satisfying the given conditions.

15. **ellipse** endpoints of major axis (3, 5) and (3, -9)
foci $(3, -2+\sqrt{45})$ and $(3, -2-\sqrt{45})$



$a = 7$

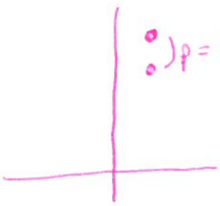
$c = \sqrt{45}$

$a^2 = b^2 + c^2 \quad 49 - 45 = b^2 \quad b = 2$

$$\boxed{\left(\frac{x-3}{2}\right)^2 + \left(\frac{y+2}{7}\right)^2 = 1}$$

$$\boxed{x = 3 + 2\cos T \quad y = -2 + 7\sin T}$$

16. **parabola** vertex (1, 8) focus $(1, 7\frac{1}{4})$

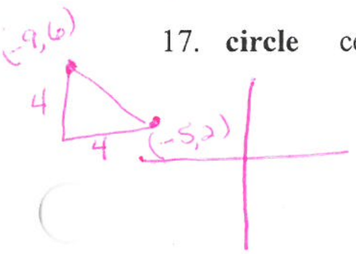


$$\boxed{y = -\frac{1}{3}(x-1)^2 + 8}$$

$a = \frac{1}{4\left(\frac{3}{4}\right)} = \frac{1}{3}$

$$\boxed{\begin{array}{l} y = -\frac{1}{3}T^2 + 8 \\ x = T + 1 \end{array}}$$

17. **circle** center (-5, 2) passes through (-9, 6)



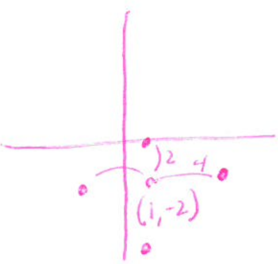
$4^2 + 4^2 = r^2$

$\sqrt{32} = r$

$$\boxed{\left(\frac{x+5}{\sqrt{32}}\right)^2 + \left(\frac{y-2}{\sqrt{32}}\right)^2 = 1} \text{ or } \boxed{(x+5)^2 + (y-2)^2 = 32}$$

$$\boxed{x = -5 + \sqrt{32}\cos T \quad y = 2 + \sqrt{32}\sin T}$$

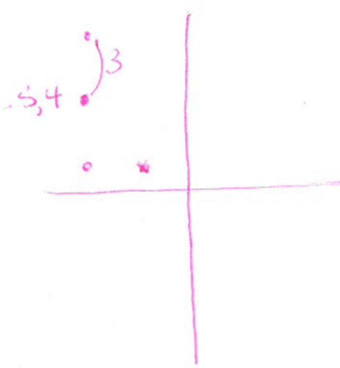
18. **ellipse** endpoints of major axis (5, -2) and (-3, -2)
endpoint of minor axis (1, 0) and (1, -4)



$$\boxed{\left(\frac{x-1}{4}\right)^2 + \left(\frac{y+2}{2}\right)^2 = 1}$$

$$\boxed{\begin{array}{l} x = 1 + 4\cos T \\ y = -2 + 2\sin T \end{array}}$$

19. **hyperbola** vertices (-5, 1) and (-5, 7) foci $(-5, 4+\sqrt{13})$ and $(-5, 4-\sqrt{13})$



$$\boxed{-\left(\frac{x+5}{2}\right)^2 + \left(\frac{y-4}{3}\right)^2 = 1}$$

$$\boxed{\begin{array}{l} x = -5 + 2\tan T \\ y = 4 + 3\sec T \end{array}}$$

$c = \sqrt{13}$

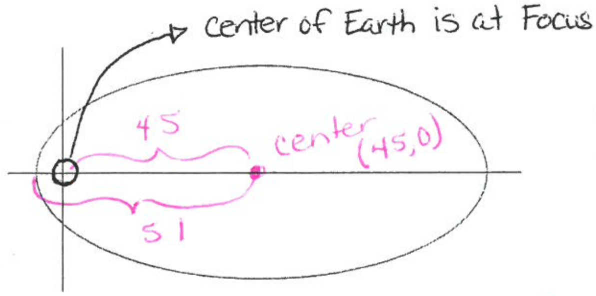
$a = 3$

$c^2 = a^2 + b^2$

$13 - 9 = b^2$

$2 = b$

20. A satellite is in elliptical orbit around Earth as shown in the picture. The major radius of the ellipse is 51 thousand miles, and the focal radius is 45 thousand miles. The center of Earth is at one focus of the ellipse.



$a = 51$

$c = 45$

$a^2 = b^2 + c^2$

$51^2 - 45^2 = b^2$

$b = 24$

answers in thousands

a) What is the center of the ellipse? $(45, 0)$

b) What is the minor radius? 24

c) What is the eccentricity? $\frac{45}{51}$

d) Write the equation of the ellipse. $\boxed{\left(\frac{x-45}{51}\right)^2 + \left(\frac{y}{24}\right)^2 = 1}$ $\boxed{\begin{array}{l} x = 45 + 51\cos T \\ y = 0 + 24\sin T \end{array}}$

e) The satellite is closest to Earth when it is at a vertex of the ellipse. The radius of Earth is about 4000 miles. What is the closet the satellite comes to the surface of Earth?

satellite Earth

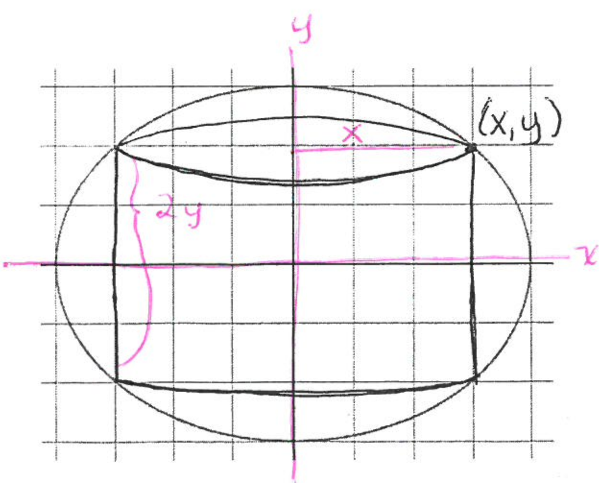
6000 miles

6000 dist.

$-$ 4000 radius

$\boxed{2000 \text{ miles}}$

21. The figure shows the ellipsoid formed by rotating the graph of $9x^2 + 16y^2 = 144$ about the y-axis. A cylinder is inscribed in the ellipsoid, with its axis along the y-axis and its two bases touching the ellipsoid. Find the value of x that maximizes the volume of the cylinder.



x = 3.27 (radius)

Max volume = 116.08

altitude = 3.46

$V = \pi r^2 h$

$V = \pi x^2 2y$

$V = \pi x^2 \cdot 2\left(\sqrt{\frac{(144-9x^2)}{16}}\right)$

max: $(3.27, 116.08)$

$9x^2 + 16y^2 = 144$

$y = \sqrt{\frac{144-9x^2}{16}}$