Section 4-3 Activity

Prove that the given equation is an identity.

1. $sin^3\theta cos^2\theta = sin^3\theta - sin^5\theta$

$= \sin^3\theta(1-\sin^2\theta)$

$\sin^3\theta\cos^2\theta = \sin^3\theta \cdot \cos^2\theta$

2. $tan^2\theta cos^2\theta + cot^2\theta sin^2\theta = 1$

$\frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{\cos^2\theta}{1} + \frac{\cos^2\theta}{\sin^2\theta} \cdot \frac{\sin^2\theta}{1} =$

$\sin^2\theta + \cos^2\theta =$

$1 = 1$

3. $\frac{1-\sin v}{\cos v} + \frac{\cos v}{1-\sin v} = 2\sec v$

$\frac{(1-\sin v)}{(1-\sin v)} \cdot \frac{1-\sin v}{\cos v} + \frac{\cos v}{1-\sin v} \cdot \frac{\cos v}{\cos v}$

$\frac{(1-\sin v)(1-\sin v)}{(1-\sin v)(\cos v)} + \frac{\cos v^2}{(1-\sin v)(\cos v)} =$

$\frac{1-2\sin v+\sin^2 v+\cos^2 v}{(1-\sin v)(\cos v)} =$

$\frac{1-2\sin v+1}{(1-\sin v)(\cos v)} =$

$\frac{2-2\sin v}{(1-\sin v)(\cos v)} =$

$\frac{2(1-\sin v)}{(1-\sin v)(\cos v)} =$

$\frac{2}{\cos v} =$

$\frac{2}{\cos v} =$

$2 \cdot \frac{1}{\cos v} =$

$2\sec v = 2\sec v$

4. $\sin\theta(\cot\theta + \tan\theta) = \sec\theta$

$$\frac{\sin\theta}{1}\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right) =$$

$$\frac{\sin\theta}{1}\cdot\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{1}\cdot\frac{\sin\theta}{\cos\theta} =$$

$$\frac{\cos\theta}{\cos\theta}\cdot\frac{\cos\theta}{1} + \frac{\sin^2\theta}{\cos\theta} =$$

$$\frac{\cos^2\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta} =$$

$$\frac{1}{\cos\theta}$$

$\sec\theta = \sec\theta$

5. $(3 + 2\sin\theta)(3 - 2\sin\theta) = 4\cos^2\theta + 5$

$$9 - 6\sin\theta + 6\sin\theta - 4\sin^2\theta =$$

$$9 - 4\sin^2\theta =$$

$$9 - 4(1 - \cos^2\theta) =$$

$$9 - 4 + 4\cos^2\theta =$$

$$5 + 4\cos^2\theta =$$

$4\cos^2\theta + 5 = 4\cos^2\theta + 5$

6. $\frac{(1+\cos\theta)}{(1+\cos\theta)}\frac{1}{1} - \frac{\sin^2\theta}{1+\cos\theta} = \cos\theta$

$$\frac{1 + \cos\theta - \sin^2\theta}{1 + \cos\theta} =$$

$$\frac{1 + \cos\theta - (1 - \cos^2\theta)}{1 + \cos\theta} =$$

$$\frac{\cos\theta + \cos^2\theta}{1 + \cos\theta} =$$

$$\frac{\cos\theta(1 + \cos\theta)}{1 + \cos\theta} =$$

$\cos\theta = \cos\theta$