Section 1-4
Composition of Functions

Definition: Given two functions f and g, the composite function, denoted by f o g (read as "f composed with g"), is defined by $(f \circ g)(x) = f(g(x))$

For the given functions f and g $\quad f(x) = 2x + 1 \quad$ $g(x) = 3x^2 - 5$

Find:

1. f(g(4))

$g(4) = 3(4)^2 - 5$
$= 43$
$f(g(4)) = 2(43) + 1$
$= \boxed{87}$

2. g(f(4))

$f(4) = 2(4) + 1$
$= 9$
$g(f(4)) = 3(9^2) - 5$
$= \boxed{238}$

3. f(f(2))

$f(2) = 2(2) + 1$
$= 5$
$f(f(2)) = 2(5) + 1$
$= \boxed{11}$

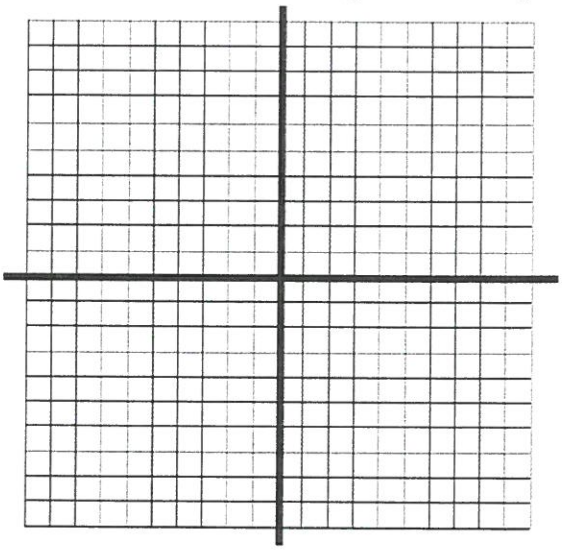
4. g(g(0))

$g(0) = 3(0^2) - 5$
$= -5$
$g(g(0)) = 3(-5^2) - 5$
$\boxed{70}$

If the domains of two functions are restricted, then the domain and range of the composition of those two functions are also restricted.

$g(x) = x - 3 \quad$ for $2 \le x \le 7$

$f(x) = -2x + 8 \quad$ for $1 \le x \le 5$



a. find f(g(5)) $\quad g(5) = 5 - 3 = 2$
$f(g(5)) = -2(2) + 8$
$\boxed{4}$

b. find f(g(8)) $\quad g(8) = \emptyset$
8 not in domain of g

c. find f(g(2)) $\quad g(2) = 2 - 3 = -1$
-1 not in domain of f $\quad f(g(2)) = \emptyset$

d. Find an equation that expresses f(g(x)) explicitly in terms of x. Find the domain and range. [put eq. for g(x) into f(x)]

$f(g(x)) = -2(x - 3) + 8$
$-2x + 6 + 8$
$= \boxed{-2x + 14}$

Dom: ① write down dom. of inner funct
② put inner funct into domain of outer function & simplify
③ find intersection of #1 & #2

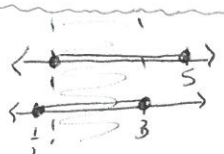
e. Find the domain of g(f(x))

① $1 \le x \le 5$

② $\frac{2}{-8} \le \frac{-2x + 8}{-8} \le \frac{7}{-8}$
$\frac{-6}{-2} \le \frac{-2x}{-2} \le \frac{-1}{-2}$
$3 \ge x \ge \frac{1}{2}$

* flip ineq signs when div. by neg

③ $1 \le x \le 5$
$\frac{1}{2} \le x \le 3$



$\boxed{1 \le x \le 3}$

① $2 \le x \le 7$

② $\frac{2}{+3} \le \frac{x - 3}{+3} \le \frac{5}{+3}$
$4 \le x \le 8$

③ $2 \le x \le 7$
$4 \le x \le 8$

dom. of comp: $4 \le x \le 7$