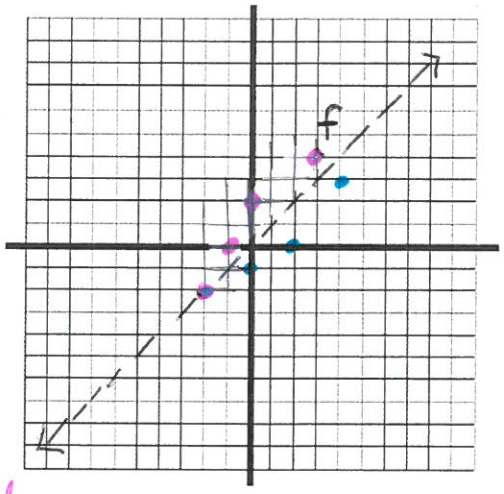
Section 1-5
One-to-One Functions and Inverse Functions

Inverse functions f and f^{-1} undo each other. For example, suppose in f the input x = 5 corresponds to the output y = 10. In the inverse function, f^{-1}, the input x = 10 would correspond to the output y =5. The x and y's are switched.

1. Given f(x) = (-2, -2), (-1, 0), (0, 2), (3, 4)
   Graph $f^{-1}(x)$ $(-2,-2),(0,-1),(2,0),(4,3)$



*Note: The inverse is reflected about the line
* shown with dashed line $y = x$

* DO NOT CONNECT when given ordered pairs. Doing so means all decimal values of x are included, and they are NOT because there's no equation.

Write an equation for the inverse by interchanging the variables and solving for y. Then, graph both f and f^{-1} on the same screen. (you can graph y = x, too) Is the inverse a function?

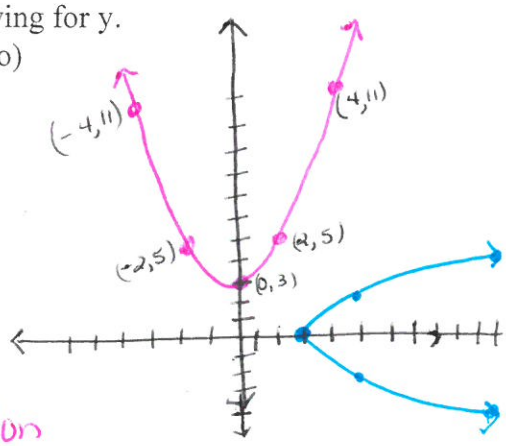
2. $y = \frac{1}{2}x^2 + 3$

$x = \frac{1}{2}y^2 + 3$

$x - 3 = \frac{1}{2}y^2$

$2(x-3) = y^2$

$\pm\sqrt{2(x-3)} = y^{-1}$



* inverse is a relation
* use only upper branch if want it to be a function

*There is a simpler way to plot the inverse of a function using **parametric** equations.

original → f(x): $X_1 = T$, $Y_1 = \frac{1}{2}T^2 + 3$

inverse → $f^{-1}(x)$: $X_2 = \frac{1}{2}T^2 + 3$, $Y_2 = T$

*** just switch x & y to graph inverse in parametric mode

One-to-One Function Each x in the domain has one and only one image in the range

- no y's can repeat, or the original function f(x) passes the *horizontal line test*)
- Strictly increasing or strictly decreasing functions are one-to-one functions
- One-to-One Functions are considered *invertible*

Vertical line test helps determine a function ⇒ horizontal line test helps confirm it's a 1 to 1 function.

3. Suppose you have fixed costs of owning a car (car payments, insurance, etc) of \$300 per month and operating costs of \$0.25 per mile you drive. The monthly cost of owning the car is given by the linear function c(x) = 0.25x + 300 where x is the number of miles you drive and c is the number of dollars per month you spend.

a. Find c(1000). Explain the real-world meaning of the answer.

$c(1000) = 0.25(1000) + 300$

$= \$550$

The cost is \$550 if you drive 1000 miles.

b. Find $c^{-1}(x)$. What does this new equation mean?

$y = 0.25x + 300$

$x = 0.25y + 300$

$x - 300 = 0.25y$

$4(x - 300) = y$