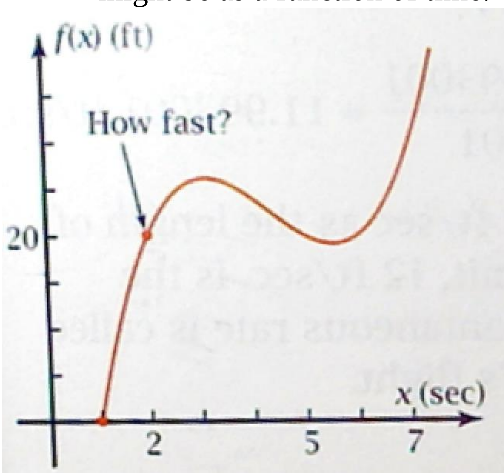
Notes 15.5 Instantaneous Rate of Change: The Derivative

1. Suppose that a bird takes off from the ground at time x= 1 second. It climbs for a while, then dives for a while, and then swoops back up again. The figure shows what its height might be as a function of time.



From the graph you can tell that the bird still climbing at x = 2 sec. The question is, "At what *rate* is the bird climbing at the instant x = 2?"

You will learn how to calculate the **derivative** of certain kinds of functions, which tells you the **instantaneous rate.**

Suppose the bird's height is given by the function $f(x) = x^3 - 13x^2 + 52x - 40$

$$f'(x) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$$

The **instantaneous rate of change** of f(x) at x = c is called the **derivative** and is denoted $f'(x)$, which is pronounced "f prime of x". It is equal to the **limit** of the average rate as x approaches c.

$$f'(x) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

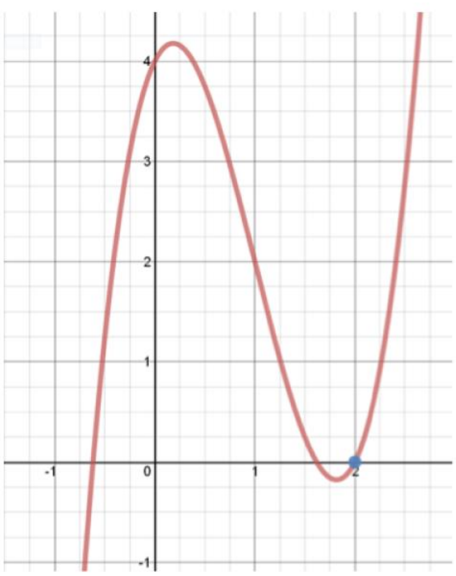
2. $f(x) = 2x^3 - 6x^2 + 2x + 4$

Find the instantaneous rate of change at x = 2

f(2) =

$$f'(x) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$$

The value of the derivative of f(x) at x = c equals the **slope of the tangent line** to the graph of f at x = c.



Find the equation of the tangent line at x = 2.

Extra example: Find the instantaneous rate of change at x = 1.