

Section 3-6 Inverse Circular Relations (Arccosine)

note: Different

* \cos^{-1} means just the principal value
 * arccos means all general solutions

{ values between 0° & 180°
 or 0π & π }

Principal Solution

$$\cos^{-1} \frac{\sqrt{3}}{2}$$

$$x \approx 0.5236 \text{ rads}$$

$$\theta = 30^\circ$$

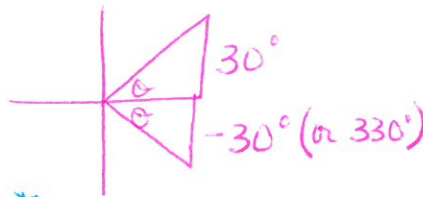
(Now that you see $\theta = 30^\circ$, the exact radian value is $\frac{\pi}{6}$)

All Solutions

$$\arccos \frac{\sqrt{3}}{2}$$

→ this is a ratio. cosine ratio is positive in Quad. I + IV

Find first five positive values.



* First 5 pos. values

General Sol:

$$\pm 30^\circ + 360^\circ n$$

$$\text{OR}$$

$$\pm \frac{\pi}{6} + 2\pi n$$

Specific Sol:

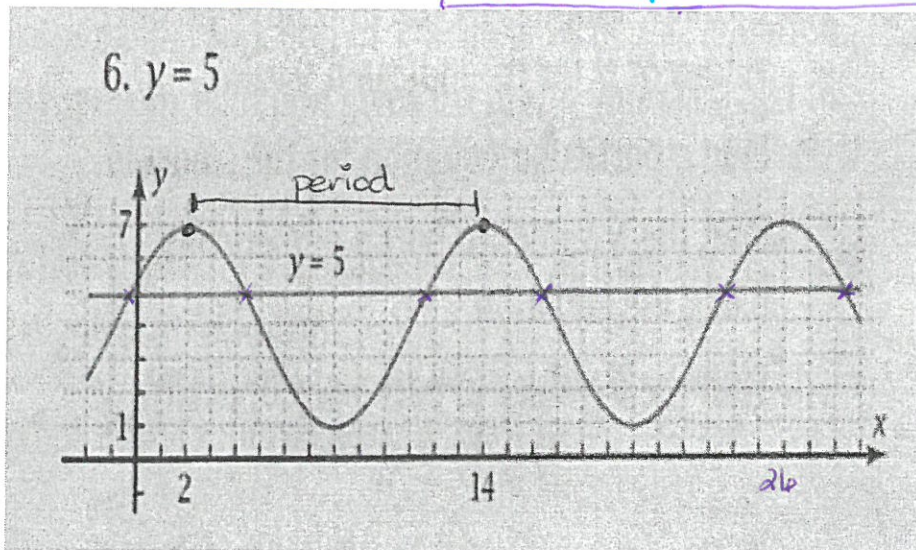
$$-30^\circ, 330^\circ, 690^\circ, 1050^\circ$$

$$* 30^\circ, 390^\circ, 750^\circ, 1110^\circ$$

OR

$$-\frac{\pi}{6}, \frac{11\pi}{6}, \frac{23\pi}{6}, \frac{35\pi}{6}$$

$$* \frac{\pi}{6}, \frac{13\pi}{6}, \frac{25\pi}{6}, \frac{37\pi}{6}$$



a. Estimate **graphically** the x-values shown for the indicated y-value.

$$-0.3, 4.3, 11.6, 16.3, 23.6, 28.3$$

b. Find a particular equation of the sinusoid.

S.A. $\rightarrow 4$ (mdpt. of high + low)

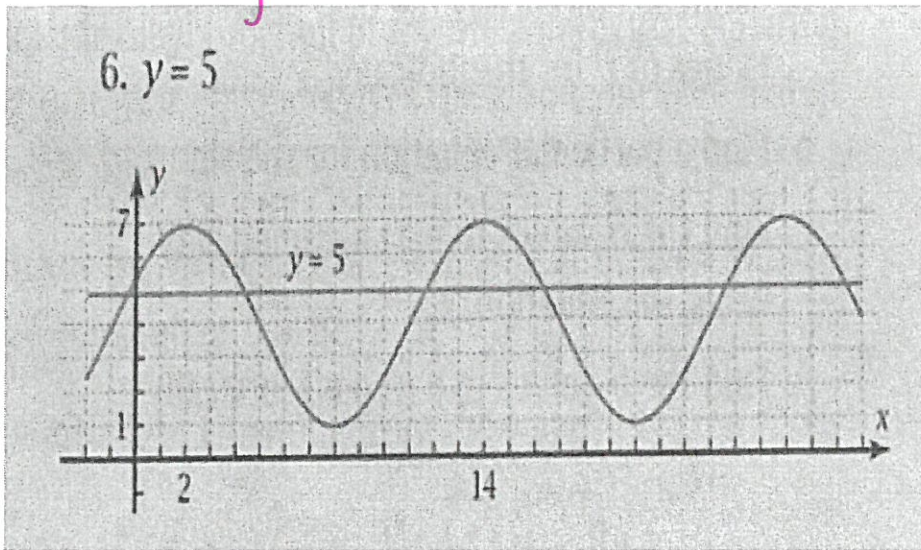
Amp $\rightarrow 3$

Ph. disp $\rightarrow 2$ (for cosine)

Period $\rightarrow 12$ $2\pi * hd = 12$ $hd = \frac{12}{2\pi} = \frac{6}{\pi}$

$$y = 4 + 3 \cos \frac{\pi}{6} (x - 2)$$

$$y = 4 + 3 \cos \frac{\pi}{6} (x-2)$$



c. Find the x-values in part (a) **numerically**. (**Use intersect feature on Calculator)

$$-0.35, 4.35, 11.65, 16.35, 23.65, 28.35$$

d. Find the x-values in part (a) **algebraically**.

* plug 5 in for y

$$5 = 4 + 3 \cos \frac{\pi}{6} (x-2)$$

$$\frac{1}{3} = \frac{3 \cos \frac{\pi}{6} (x-2)}{3}$$

arccos $\frac{1}{3} = \cos \frac{\pi}{6} (x-2)$

* use arccos because we want multiple solutions

$$\arccos \left(\frac{1}{3} \right) = \frac{\pi}{6} (x-2)$$

* arccos $\Rightarrow \pm \cos^{-1} x + 2\pi n$

$$\pm \cos^{-1} \left(\frac{1}{3} \right) + 2\pi n = \frac{\pi}{6} (x-2)$$

$$\frac{6}{\pi} \left[\pm \cos^{-1} \left(\frac{1}{3} \right) + 2\pi n \right] = (x-2) + 2$$

$$2 + \frac{6}{\pi} \left[\pm \cos^{-1} \left(\frac{1}{3} \right) + 2\pi n \right] = x$$

General Solution

* dist. $\frac{6}{\pi}$ into brackets

* notice this is the period!

Specific Solutions

$$2 + \frac{6}{\pi} \left[\pm 1.23 + 2\pi n \right] = x$$

$$2 \pm 2.35 + 12n = x$$

$$4.35 + 12n \quad \text{AND} \quad -0.35 + 12n$$

$\rightarrow -0.35, 11.65, 23.65, 35.65$ * match above
 $\rightarrow 4.35, 16.35, 28.35, 40.35$