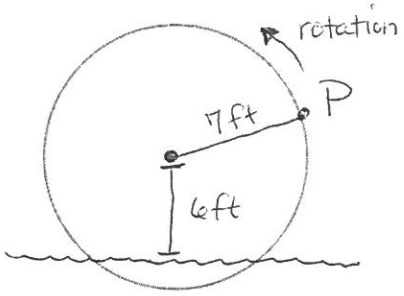
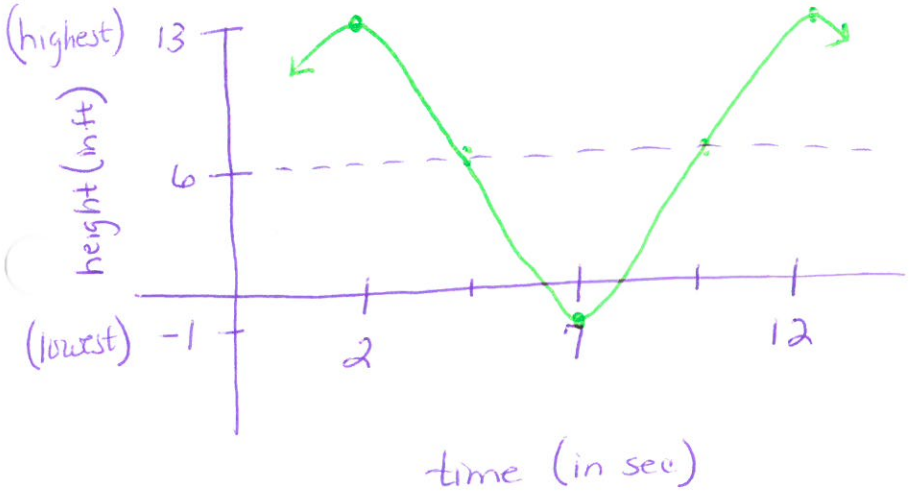
Section 3-7 Sinusoidal Functions as Mathematical Models

Waterwheel Problem:
Suppose that a waterwheel with radius of 7 feet rotates at 6 revolutions per minute (rpm). 2 seconds after you start a stopwatch, point P on the rim of the wheel is at its greatest height, d = 13 ft, above the surface of the water. The center of the waterwheel is 6 ft above the surface.



a) Sketch the graph of d as a function of t, in seconds, since you started the stopwatch.



* if 6 rev. per min, each revolution takes 10 seconds (that's the period)

$2\pi * hd = 10 \quad hd = \frac{10}{2\pi}$

$hd = \frac{5}{\pi}$

b) Write an equation of the sinusoid.

** use cosine since arccos is what we've studied

$y = 6 + 7\cos\frac{\pi}{5}(x-2)$

c) How high above or below the water's surface will P be at time t = 17.5 sec?

time = x so → $y = 6 + 7\cos\frac{\pi}{5}(17.5-2)$ $y = -0.6574$ ft.

d) At what time t was point P first emerging from the water?

ft = 0 → so, y = 0

① Graphically → find intersection of y_1 → equation and y_2 → 0 at point where it's first emerging (1st pos. time after it's under water -- after y is neg)

② Numerically → use table to find same thing as in #1 (1st pos. time...)

* not very accurate.

③ Algebraically → set y = 0

$0 = 6 + 7\cos\frac{\pi}{5}(x-2)$

$-\frac{6}{7} = \cos\frac{\pi}{5}(x-2)$

$\pm\cos^{-1}\left(-\frac{6}{7}\right) + 2\pi n = \frac{\pi}{5}(x-2)$

$2 + \frac{5}{\pi}\left[\pm\cos^{-1}\left(-\frac{6}{7}\right) + 2\pi n\right] = x$ * Gen. Sol.

$2 \pm 4.14 + 10n$

$6.14 + 10n$; 6.14, 16.14 6.14 sec going in

or

$-2.14 + 10n$; -2.14, 7.86 coming out