Prove that the given equation is an identity.

1. $\tan x(\cot x + \tan x) = sec^2x$

$\tan x \cdot \cot x + \tan x \cdot \tan x =$

$\tan x \left(\frac{1}{\tan x}\right) + \tan^2 x =$

$1 + \tan^2 x =$

$\boxed{\sec^2 x = \sec^2 x}$

2. $cos^2\theta + tan^2\theta cos^2\theta = 1$

$\cos^2\theta + \frac{\sin^2\theta}{\cancel{\cos^2\theta}} \cdot \frac{\cancel{\cos^2\theta}}{1} =$

$\cos^2\theta + \sin^2\theta =$

$\boxed{1 = 1}$

3. $(5\cos x - 4\sin x)^2 + (4\cos x + 5\sin x)^2 = 41$

$(5\cos x - 4\sin x)(5\cos x - 4\sin x) + (4\cos x + 5\sin x)(4\cos x + 5\sin x)$

$25\cos^2 x - 20\sin x\cos x - 20\sin x\cos x + 16\sin^2 x$
$+ 16\cos^2 x + 20\sin x\cos x + 20\sin x\cos x + 25\sin^2 x$

$25\cos^2 x + 16\sin^2 x + 16\cos^2 x + 25\sin^2 x =$ (sinx cosx terms cancel out)

$41\cos^2 x + 41\sin^2 x =$

$41(\cos^2 x + \sin^2 x) =$

$41(1) =$

$\boxed{41 = 41}$

4. $\frac{\sec x+1}{\sec x+1} \cdot \frac{\cos x}{\sec x - 1} - \frac{\cos x}{tan^2 x} = cot^2 x$

$$\frac{\cos x(\sec x+1)}{(\sec x+1)(\sec x-1)} - \frac{\cos x}{\tan^2 x} =$$

$$\frac{\cos x \cdot \sec x + \cos x}{\sec^2 x - 1} - \frac{\cos x}{\tan^2 x} =$$

$$\frac{\frac{\cos x}{1} \cdot \frac{1}{\cos x} + \cos x}{\tan^2 x} - \frac{\cos x}{\tan^2 x}$$

$$\frac{1 + \cos x}{\tan^2 x} - \frac{\cos x}{\tan^2 x}$$

$$\frac{1}{\tan^2 x} =$$

$$\boxed{\cot^2 x = \cot^2 x}$$

5. $\frac{(\sec\theta - \tan\theta)}{\sec\theta - \tan\theta} \cdot \frac{\sec\theta + tan\theta}{1} = \frac{1}{\sec\theta - \tan\theta}$

$$\frac{\sec^2\theta + \sec\theta\tan\theta - \sec\theta\tan\theta - \tan^2\theta}{\sec\theta - \tan\theta}$$

$$\frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta}$$

$$\frac{1 + \tan^2\theta - \tan^2\theta}{\sec\theta - \tan\theta}$$

$$\frac{1}{\sec\theta - \tan\theta} = \frac{1}{\sec\theta - \tan\theta}$$