Section 4-3 Trigonometric Identities

First let's learn how to simplify trig expressions:

* Start w/ this

* work with ONLY one side

1. A) Write in terms of sine and cosine

$$\frac{\cot\theta}{\csc\theta}$$

- $\frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$
- $\frac{\cos\theta}{\sin\theta} * \frac{\sin\theta}{1}$
- $\cos\theta$

B) Show $\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$ by multiplying the fraction by $1 - \sin\theta$

- $\frac{(1-\sin\theta)}{(1-\sin\theta)}\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$
- $\frac{(1-\sin\theta)\cos\theta}{1-\sin^2\theta} =$
- $\frac{(1-\sin\theta)\cos\theta}{\cos^2\theta} =$
- $\frac{1-\sin\theta}{\cos\theta} = \frac{1-\sin\theta}{\cos\theta}$

* Comm. Denom is: $\sin u \cdot \cos u$

C) Get a common denominator

$$\frac{1+\sin u}{\sin u} + \frac{\cot u - \cos u}{\cos u}$$

- $\frac{\cos u(1+\sin u)}{\cos u \sin u} + \frac{\sin u(\cot u - \cos u)}{\sin u \cos u}$
- $\frac{\cos u + \cos u \sin u + \sin u\left(\frac{\cos u}{\sin u}\right) - \sin u \cos u}{\sin u \cos u}$
- $\frac{\cos u + \cos u}{\sin u \cos u}$
- $\frac{2\cos u}{\sin u \cos u}$
- $\frac{2}{\sin u}$
- $2\csc u$

D) By Factoring

$$\frac{\sin^2 u - 1}{\tan u \sin u - \tan u}$$

* Diff of squares
* GCF
* etc.

- $\frac{(\sin u - 1)(\sin u + 1)}{\tan u(\sin u - 1)}$
- $\frac{\sin u + 1}{\tan u}$

Show the steps to transform the left side to the right side as shown.

2. $\csc\theta \cdot \tan\theta$ to $\sec\theta$

- $\frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta}$
- $\frac{1}{\cos\theta}$
- $\sec\theta$

3. $\sec x - \cos x$ to $\sin x \tan x$

- $\frac{1}{\cos x} - \frac{\cos x}{1}$
- $\frac{1}{\cos x} - \frac{\cos x}{1}\left(\frac{\cos x}{\cos x}\right)$
- $\frac{1 - \cos^2 x}{\cos x}$
- $\frac{\sin^2 x}{\cos x}$
- $\frac{\sin x \cdot \sin x}{\cos x}$
- $\sin x \cdot \frac{\sin x}{\cos x}$
- $\boxed{\sin x \cdot \tan x}$

4. $(\sec\theta - 1)(\sec\theta + 1)$ to $\tan^2\theta$

- $\sec^2\theta - \sec\theta + \sec\theta - 1$
- $\sec^2\theta - 1$
- $\boxed{\tan^2\theta}$

5. $\frac{1-\cos^2 x}{\tan x}$ to $\sin x \cos x$

- $\frac{\sin^2 x}{\frac{\sin x}{\cos x}}$
- $\frac{\sin^2 x}{1} \cdot \frac{\cos x}{\sin x}$
- $\boxed{\sin x \cdot \cos x}$

6. $\frac{\csc\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$ to $\tan\theta$

- $\left(\frac{\sin\theta}{\sin\theta}\right)\frac{\csc\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}\left(\frac{\cos\theta}{\cos\theta}\right)$
- $\frac{\sin\theta\csc\theta}{\sin\theta\cos\theta} - \frac{\cos^2\theta}{\sin\theta\cos\theta}$
- $\frac{\sin\theta\left(\frac{1}{\sin\theta}\right) - \cos^2\theta}{\sin\theta\cos\theta}$
- $\frac{1 - \cos^2\theta}{\sin\theta\cos\theta}$
- $\frac{\sin^2\theta}{\sin\theta\cos\theta}$
- $\frac{\sin\theta}{\cos\theta}$
- $\boxed{\tan\theta}$

* there are mult ways to do this and other problems