

Section 4-4 Solving Trig Equations

Solve the equation in the given domain.

1. $2\cos\theta + \sqrt{3} = 0$ $\theta \in [0^\circ, 720^\circ]$

$$2\cos\theta = -\sqrt{3}$$

$$\cos\theta = \frac{-\sqrt{3}}{2}$$

Arccos \rightarrow

$$\theta = \pm \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + 360n$$

$$\theta = \pm 150^\circ + 360n$$

$$150^\circ, 510^\circ, 870^\circ$$

$$-150^\circ, 210^\circ, 570^\circ, 930^\circ$$

$$150^\circ, 210^\circ, 510^\circ, 570^\circ$$

* in 2nd (Princ.) and 3rd (Secondary) quadrants

2. $\tan(\theta - 81^\circ) = 1$ $\theta \in [-180^\circ, 540^\circ]$

arctan \rightarrow $\theta - 81 = \tan^{-1}(1) + 180n$

$$\theta = 81 + \tan^{-1}(1) + 180n$$

$$\theta = 81 + 45 + 180n$$

$$\theta = 126 + 180n$$

$$-54^\circ, 126^\circ, 306^\circ, 486^\circ$$

$$-234, -54, 126, 306, 486, 666$$

3. $10\sin(x - 0.2) = -3$ for x in the domain $[0, 4\pi]$ ≈ 12.57

$$\sin(x - 0.2) = \frac{-3}{10}$$

arcsin \rightarrow $\begin{cases} x - 0.2 = \sin^{-1}\left(\frac{-3}{10}\right) + 2\pi n \\ \text{AND } x - 0.2 = \pi - \sin^{-1}\left(\frac{-3}{10}\right) + 2\pi n \end{cases}$

$$3.65, 6.18, 9.93, 12.47$$

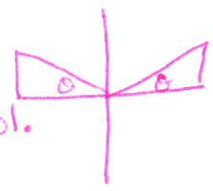
AND $\begin{cases} x - 0.2 = -0.30 + 2\pi n \\ x - 0.2 = 3.45 + 2\pi n \end{cases}$

AND $\begin{cases} x = -0.10 + 2\pi n \\ x = 3.65 + 2\pi n \end{cases}$

$$-0.10, 6.18, 12.47$$

$$3.65, 9.93, 16.22$$

* $\sin \theta$ is pos in I+II,
 so θ AND $180-\theta$ are sol.



4. $5\sin \pi x = 2$ $x \in [-2, 4]$

arc sin \rightarrow

$$\sin \pi x = \frac{2}{5}$$

$$\begin{cases} \pi x = \sin^{-1}\left(\frac{2}{5}\right) + 2\pi n \\ \pi x = \pi - \sin^{-1}\left(\frac{2}{5}\right) + 2\pi n \end{cases}$$

$$\begin{cases} x = \frac{\sin^{-1}\left(\frac{2}{5}\right)}{\pi} + 2n \\ x = \frac{\pi - \sin^{-1}\left(\frac{2}{5}\right)}{\pi} + 2n \end{cases}$$

$$\begin{cases} x = 0.13 + 2n \\ x = 0.87 + 2n \end{cases}$$

$$\begin{cases} -1.87, 0.13, 2.13, 4.13 \\ -1.13, 0.87, 2.87, 4.87 \end{cases}$$

$-1.87, -1.13, 0.13, 0.87, 2.13, 2.87$

5. $\cos^2 \theta + \sin \theta + 1 = 0$ $\theta \in [-90^\circ, 270^\circ]$

* must have same vars.
 (try functions)

think of this
 as $x^2 - x + 2$
 $(x+1)(x-2)$

$$(1 - \sin^2 \theta) + \sin \theta + 1 = 0$$

$$-\sin^2 \theta + \sin \theta + 2 = 0$$

$$\sin^2 \theta - \sin \theta + 2 = 0$$

$$(\sin \theta + 1)(\sin \theta - 2) = 0$$

$\sin \theta + 1 = 0$
 $\sin \theta - 2 = 0$ } Now we have 2 problems to solve

- $\sin \theta = -1$

$$\begin{cases} \theta = \sin^{-1}(-1) + 360^\circ n \\ \theta = 180 - \sin^{-1}(-1) + 360^\circ n \end{cases}$$

$\theta = -90^\circ$
 $\theta = 270^\circ$

-90°

- $\sin \theta = 2$
 $\theta = \sin^{-1}(2)$ * impossible
 $\sin \theta$ must be ≤ 1

6. $37\sin \theta + 31 = 10\cos^2 \theta$ $\theta \in [0^\circ, 720^\circ]$

$$37\sin \theta + 31 = 10(1 - \sin^2 \theta)$$

$$37\sin \theta + 31 = 10 - 10\sin^2 \theta$$

$$10\sin^2 \theta + 37\sin \theta + 21 = 0$$

$$(10\sin \theta + 7)(\sin \theta + 3) = 0$$

$$10\sin \theta + 7 = 0$$

$$10\sin \theta = -7$$

$$\sin \theta = -\frac{7}{10}$$

$\sin \theta + 3 = 0$
 $\sin \theta = -3$
 * not possible

$$\begin{cases} \theta = \sin^{-1}\left(-\frac{7}{10}\right) + 360^\circ n \\ \theta = 180 - \sin^{-1}\left(-\frac{7}{10}\right) + 360^\circ n \end{cases}$$

$$\begin{aligned} & -44.43^\circ + 360^\circ n \\ & 224.43^\circ + 360^\circ n \\ & -44.43^\circ, 315.57^\circ, 675.57^\circ \\ & 224.43^\circ, 584.43^\circ, 944.43^\circ \end{aligned}$$

$224.43^\circ, 315.57^\circ, 584.43^\circ, 675.57^\circ$

7. $0.2x + \sin x = 2$ for all real values of x

* Doesn't make sense. How can x represent an angle AND something else in the equation.

8. $4 \tan 2\theta = -5$ $\theta \in [0, 180^\circ]$

$$\tan 2\theta = -\frac{5}{4}$$

$$\arctan \rightarrow 2\theta = \tan^{-1}\left(-\frac{5}{4}\right) + 180n$$

$$\theta = \frac{\tan^{-1}\left(-\frac{5}{4}\right) + 180n}{2}$$

$$\theta = -25.67 + 90n$$

$$\boxed{64.33^\circ, 154.33^\circ}$$

* Confirm by graphing: (watch window -- you're in degrees)

$$y_1 = 4 \tan 2x$$

$$y_2 = -5$$

