Section 4-5 Parametric Equations

If two related variables x and y both depend on a third, independent variable t, the pair of equations in x and t and y and t is called a **parametric function**.

Plot the graph of the following parametric function in degree mode:

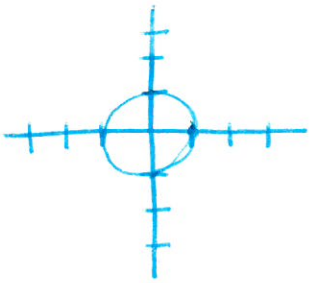
1. $x = \cos t$
   $y = \sin t$

tmin = 0
tmax = 360
t step = 5 (graphs every 5 degrees from 0 to 360)

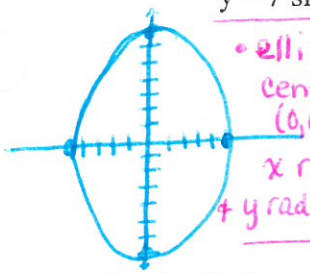
What is it a picture of? Circle with center at (0,0) & radius of 1

What would you need to change in radian mode?
- change mode to radians
- tmin = 0
- tmax = 2π
- t step = 0.1

(when choosing t step, think about how many points you want graphed between your tmin and tmax -- 0.1 will graph every tenth from 0 to 6.28...)



2. $x = 5\cos t$
   $y = 7\sin t$



- ellipse with center at (0,0) and x radius of 5 + y radius of 7

Degree mode:
tmin = 0
tmax = 360
tstep = 5

Radian mode:
tmin = 0
tmax = 2π
tstep = 0.1

Start $\begin{cases} x_{min} = -10 \\ x_{max} = 10 \\ y_{min} = -10 \\ y_{max} = 10 \end{cases}$ ⇒ adjust to see complete picture

Use the Pythagorean Property to eliminate the parameter t. (This will give you one equation in x and y)

↘ both cos t & sin t need to be squared

1. $(x)^2 = (\cos t)^2$
   $+ (y)^2 = (\sin t)^2$

$x^2 + y^2 = \cos^2 t + \sin^2 t$

$\boxed{x^2 + y^2 = 1}$

2. $x = 5\cos t$
   $y = 7\sin t$

$\left(\frac{x}{5}\right)^2 = (\cos t)^2$
$+ \left(\frac{y}{7}\right)^2 = (\sin t)^2$

$\boxed{\left(\frac{x}{5}\right)^2 + \left(\frac{y}{7}\right)^2 = 1}$

1.) get cos t and sin t alone on one side

2.) square both sides

3.) Add the two equations

4.) simplify

3. $x = 6\cos t$
   $y = 6\sin t$

$(\div 6)$

$\left(\frac{x}{6}\right)^2 = (\cos t)^2$
$+ \left(\frac{y}{6}\right)^2 = (\sin t)^2$ (square both sides)

(add equations & simplify)

$\boxed{\left(\frac{x}{6}\right)^2 + \left(\frac{y}{6}\right)^2 = 1}$

4. $x = 4 + 3\cos t$
   $y = -1 + 6\sin t$

$x - 4 = 3\cos t$
$y + 1 = 6\sin t$

$\left(\frac{x-4}{3}\right)^2 = (\cos t)^2$
$+ \left(\frac{y+1}{6}\right)^2 = (\sin t)^2$

$\boxed{\left(\frac{x-4}{3}\right)^2 + \left(\frac{y+1}{6}\right)^2 = 1}$

General parametric equation for an ellipse:

(h, k) is center of the ellipse
a is the x-radius
b is the y-radius

$$x = h + a\cos T$$

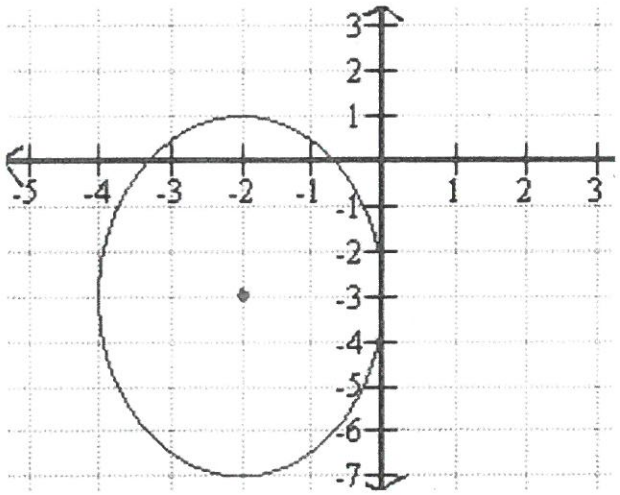
$$y = k + b\sin T$$

cos T goes with x because in the unit circle, cos θ = x coordinate.

Same reason that sin T goes with y

Write parametric equations for this ellipse.

5.



center: (-2, -3)

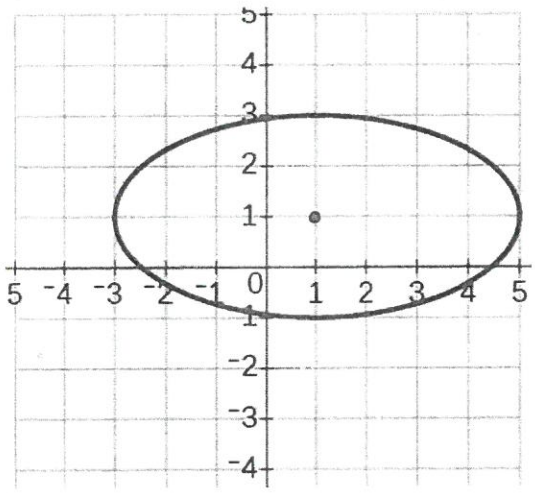
x rad: 2

y rad: 4

$x = -2 + 2\cos T$

$y = -3 + 4\sin T$

6.



center: (1, 1)

x rad: 4

y rad: 2

$x = 1 + 4\cos T$

$y = 1 + 2\sin T$