Section 5-2 Composite Argument Property for cos(A – B)

This property helps to work backwards to express cosine as a linear combination of sine and cosine. Example: If you wanted to find cos15° exactly, you could do cos(45°- 30°)

Important: Does the Distributive Property hold true for this property?

$\cos(45° - 30°) = cos45° - cos30°$???

cos15°		0.707 – 0.866
0.966	≠	-0.159

Composite Argument Property for $\cos(A - B)$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

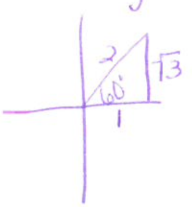
Express each equation as a linear combination of cosine and sine.

1. $y = 20\cos(\theta - 60)$

Option 1 (use formula):

$A = \theta$
$B = 60°$

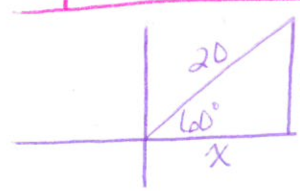
$y = 20[\cos\theta\cos 60° + \sin\theta\sin 60°]$



$y = 20\left[\cos\theta\cdot\left(\frac{1}{2}\right) + \sin\theta\cdot\left(\frac{\sqrt{3}}{2}\right)\right]$

$y = 10\cos\theta + 10\sqrt{3}\sin\theta$
OR
$y = 10\cos\theta + 17.32\sin\theta$

Option 2 (graph):



$\cos 60° = \frac{x}{20}$ $\quad x = 20\cdot\cos 60°$
$x = 10$

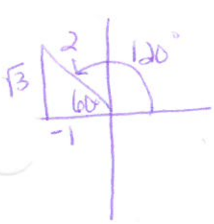
$\sin 60° = \frac{y}{20}$ $\quad y = 20\cdot\sin 60°$
$y = 17.32$

$y = 10\cos\theta + 17.32\sin\theta$

2. $y = 8\cos(2\theta - 120)$

$y = 8[\cos 2\theta\cos 120° + \sin 2\theta\sin 120°]$

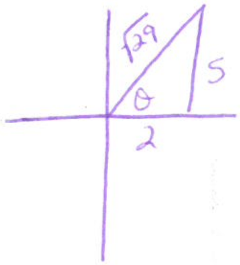
$y = 8\left[\cos 2\theta\cdot\left(-\frac{1}{2}\right) + \sin 2\theta\left(\frac{\sqrt{3}}{2}\right)\right]$



$y = -4\cos 2\theta + 4\sqrt{3}\sin 2\theta$
OR
$y = -4\cos 2\theta + 6.93\sin 2\theta$

Solve the equation. write this side as a single cosine w/ a phase displacement.

3. $\underbrace{2\cos\theta + 5\sin\theta}$ $= 4$ $\theta \in [0, 360°]$



$\tan^{-1}\left(\frac{5}{2}\right)$

$$\sqrt{29}\cos(\theta - 68.2°) = 4$$

$$\cos(\theta - 68.2°) = \frac{4}{\sqrt{29}}$$

$$\theta - 68.2° = \arccos\frac{4}{\sqrt{29}}$$

$$\theta = 68.2° \pm \cos^{-1}\left(\frac{4}{\sqrt{29}}\right) + 360°n$$

$$\theta = 68.2° \pm 42.03 + 360°n$$

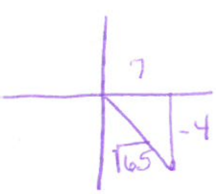
$$\theta = 110.23°, 470.23°$$

$$\theta = 26.17°, 386.17°$$

$$\boxed{\theta = 26.17°, 110.23°}$$

*** To check graphically:
① put left side in y_1
② put right side in y_2
③ put calc in correct mode
④ use correct window
⑤ find intersection(s)

4. $7\cos x - 4\sin x = 6$ $x \in [0, 2\pi]$ $2\pi \approx 6.28$



$\tan^{-1}\left(\frac{-4}{7}\right) = -0.52$ OR 5.76

either is fine

$$\sqrt{65}\cos(x + 0.52) = 6$$

$$\cos(x + 0.52) = \frac{6}{\sqrt{65}}$$

$$x + 0.52 = \arccos\left(\frac{6}{\sqrt{65}}\right)$$

$$x = -0.52 \pm \cos^{-1}\left(\frac{6}{\sqrt{65}}\right) + 2\pi n$$

$$x = -0.52 \pm 0.73 + 2\pi n$$

$$x = -1.25, 5.03$$

$$x = 0.21, 6.49$$

$$\boxed{x = 0.21, 5.03}$$

* Check on calculator