Section 5-3 Other Composite Argument Properties

* What do you notice about the graphs of even functions? odd?

even functions

$f(-x) = f(x)$

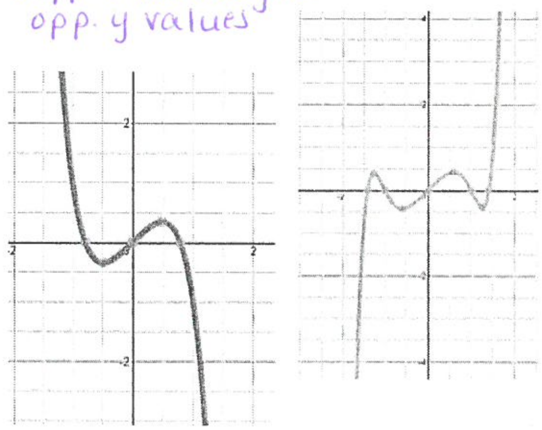
*opp x values give same y values

Reflect across y-axis

Odd functions

$f(-x) = -f(x)$

* opp x values give opp. y values

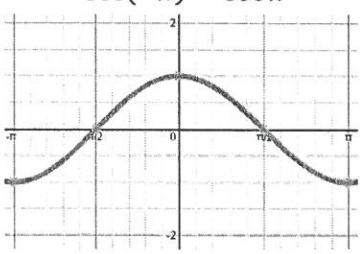


Reflect across the origin.
(Rotate 180° w/ center of rotation at the origin)

even function

$\cos(-x) = \cos x$

$\sec(-x) = \sec x$



$\cos(45) = \frac{\sqrt{2}}{2}$

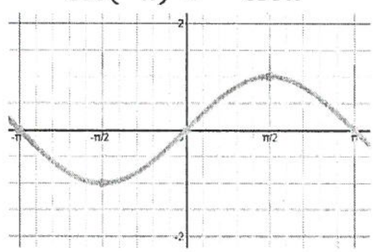
$\cos(-45) = \frac{\sqrt{2}}{2}$

* cos. of pos. ∠ is the same as the cos. of neg. ∠

odd function

$\sin(-x) = -\sin x$

$\csc(-x) = -\csc x$



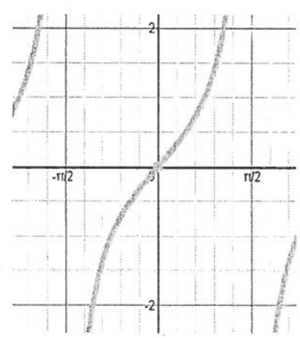
$\sin(30) = \frac{1}{2}$

$\sin(-30) = -\frac{1}{2}$

*Sine of pos ∠ is the opp. of the sine of neg ∠

odd function

$\tan(-x) = -\tan x$



$\cot(-x) = -\cot x$

So **cosine and its reciprocal, **secant**, are the only **EVEN** functions, rest of them are odd!

cofunction properties

$\cos\theta = \sin(90 - \theta)$

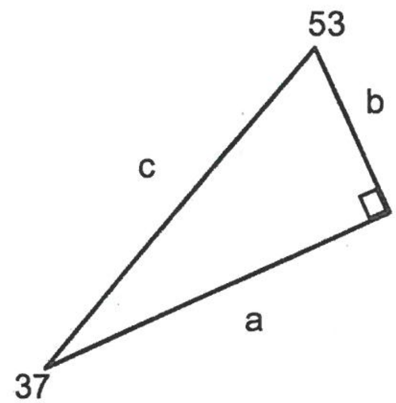
$sin\theta = \cos(90 - \theta)$

$\tan\theta = \cot(90 - \theta)$

$cot\theta = \tan(90 - \theta)$

$\sec\theta = \csc(90 - \theta)$

$csc\ \theta = \sec(90 - \theta)$



Proof:

$$\sin\theta = \cos(90° - \theta)$$

$= \cos 90° \cos\theta + \sin 90° \sin\theta$

$= (0)\cos\theta + (1)\sin\theta$

$= 0 + \sin\theta$

$\boxed{\sin\theta = \sin\theta}$

Other Composite Arguments

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Proof:

$$\sin(\theta + 30) + \cos(\theta + 60) = cos\theta$$

$[\sin\theta \cos 30 + \cos\theta \sin 30] + [\cos\theta \cos 60 - \sin\theta \sin 60] =$

$\sin\theta\left(\frac{\sqrt{3}}{2}\right) + \cos\theta\left(\frac{1}{2}\right) + \cos\theta\left(\frac{1}{2}\right) - \sin\theta\left(\frac{\sqrt{3}}{2}\right) =$

$\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta + \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta =$

$\boxed{\cos\theta = \cos\theta}$

Use the composite argument properties to solve the equation.

* can 'start' w/ either side of the equation & transform to the other side.

1. $\sin 5\theta \cos 3\theta - \cos 5\theta \sin 3\theta = \frac{1}{2}$ $\theta \in [0,360°]$

'A' $= 5\theta$

'B' $= 3\theta$

Use: $\boxed{\sin A \cos B - \cos A \sin B = \sin(A-B)}$

$\sin(5\theta - 3\theta) =$

$\sin 2\theta = \frac{1}{2}$

$2\theta = \text{arcsine}\left(\frac{1}{2}\right)$

$2\theta = \sin^{-1}\left(\frac{1}{2}\right) + 360°n$

AND $2\theta = 180 - \sin^{-1}\left(\frac{1}{2}\right) + 360°n$

$2\theta = 30° + 360°n$

AND $2\theta = 150° + 360°n$

$\theta = \frac{30°}{2} + \frac{360°n}{2}$

AND $\theta = \frac{150°}{2} + \frac{360°n}{2}$

$\theta = 15° + 180°n$ $15°, 195°, 375°$

AND $\theta = 75° + 180°n$ $75°, 255°, 435°$

$\boxed{15°, 75°, 195°, 255°}$