Section 5-5 The Sum and Product Properties

How do you solve the system of equations A + B = 10 and A – B = 2?

$A+B=10$
$A-B=2$
$2A=12$

$A=6$, so $B=4$

How would you solve $\sin 3\theta + \sin\theta = 0$? Each equation has different strategies, if you only have one equation to solve with sines or cosines, try to transform that side into a product of sines/cosines so you could solve sinA = 0 or cosA = 0 zero product property

Transform the sum or difference to a product of sines and/or cosines with positive arguments.

1. cos 56° - cos 24°

cos (A + B) - cos (A – B)

to find A take 56 + 24 and divide by 2

Remember $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

Option 1:

$A+B=56$
$A-B=24$
$2A=80$
$A=40$
so $B=16$

$\cos(A+B) - \cos(A-B) \Rightarrow$

$\cos 40^\circ\cos 16^\circ - \sin 40^\circ\sin 16^\circ$

$-[\cos 40^\circ\cos 16^\circ + \sin 40^\circ\sin 16^\circ]$

$\boxed{-2\sin 40^\circ\sin 16^\circ}$

Option 2:

* Use sum → product property

$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$A \downarrow 56^\circ$, $B \downarrow 24^\circ$

$= -2\sin\left(\frac{56+24}{2}\right)\sin\left(\frac{56-24}{2}\right)$

$\boxed{= -2\sin 40^\circ\sin 16^\circ}$

2. sin1.8 + sin6.4

Option 1:

$A+B=1.8$
$A-B=6.4$
$2A=8.2$
$A=4.1$
so $B=-2.3$

$\sin(A+B) + \sin(A-B) \Rightarrow$

$\sin(4.1)\cos(-2.3) + \cos(4.1)\sin(-2.3)$

$+ \sin(4.1)\cos(-2.3) - \cos(4.1)\sin(-2.3)$

$2\sin 4.1\cos(-2.3)$

$\boxed{2\sin(4.1)\cos(2.3)}$

* must have pos arguments

$\cos(-2.3) = \cos(2.3)$

because cosine is an even funct.

Option 2

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$= 2\sin\left(\frac{1.8+6.4}{2}\right)\cos\left(\frac{1.8-6.4}{2}\right)$

$= 2\sin(4.1)\cos(-2.3)$

$\boxed{= 2\sin(4.1)\cos(2.3)}$

Transform the product into a sum or difference of sines or cosines with positive arguments.

3. $2\cos\underset{A}{73^\circ}\sin\underset{B}{62^\circ}$ Because of the product cos· *sin* this would be sin(A+B) or sin(A – B)

* use product → sum property

$2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$2\cos 73^\circ\sin 62^\circ = \sin(73^\circ+62^\circ) - \sin(73^\circ-62^\circ)$

$= \sin(135^\circ) - \sin(11^\circ)$

$= \boxed{\sin 135^\circ - \sin 11^\circ}$

4. $2\cos\overset{A}{2} \cdot \cos\overset{B}{3}$ What has product of two cosines?

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\cos(2)\cos(3) = \cos(2+3) + \cos(2-3)$

$= \cos(5) + \cos(-1)$ $\quad * \cos(-1) = \cos(1)$

$= \boxed{\cos(5) + \cos(1)}$

Now knowing how to transform sums into products and products into sums, we can use those strategies to solve equations.

*change to product so we can use the zero product property

5. $\sin 3\theta + \sin\theta = 0 \quad \theta \in [0,360°]$

$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$x \to 3\theta$, $y \to \theta$

$2\sin\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right) = 0$

* 3 factors that could = 0

$2\sin(2\theta)\cos(\theta) = 0$

① $2 = 0$ | ② $\sin(2\theta) = 0$ | ③ $\cos\theta = 0$

$\emptyset$ | $\sin 2\theta = 0$

$2\theta = \arcsin(0)$

$\begin{cases} 2\theta = \sin^{-1}(0) + 360°n \\ 2\theta = (180 - \sin^{-1}0) + 360°n \end{cases}$

$\begin{cases} 2\theta = 0° + 360°n \\ 2\theta = 180° + 360°n \end{cases}$

$\begin{cases} \theta = 0° + 180°n \\ \theta = 90° + 180°n \end{cases}$

$\theta = \arccos(0)$

$\theta = \pm\cos^{-1}(0) + 360°n$

$\theta = \pm 90° + 360°n$

$\boxed{\theta = 90°, 270°}$ $\leftarrow -90° + 360°$

$\boxed{\begin{cases} \theta = 0°, 180°, 360° \\ \theta = 90°, 270° \end{cases}}$

6. $\cos 5x - \cos x = 0 \quad x \in [0,2\pi]$

$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

$x \to 5x$, $y \to x$

$-2\sin\left(\frac{5x+x}{2}\right)\sin\left(\frac{5x-x}{2}\right) = 0$

* 3 factors that could = 0

$-2\sin(3x)\sin(2x) = 0$

① $-2 = 0$ | ② $\sin 3x = 0$

$\emptyset$ | $3x = \arcsin(0)$

$\begin{cases} 3x = \sin^{-1}(0) + 2\pi n \\ 3x = (\pi - \sin^{-1}0) + 2\pi n \end{cases}$

$\begin{cases} 3x = 0\pi + 2\pi n \\ 3x = \pi + 2\pi n \end{cases}$

$\begin{cases} x = 0\pi + \frac{2}{3}\pi n \\ x = \frac{\pi}{3} + \frac{2}{3}\pi n \end{cases}$

$\boxed{\begin{cases} x = 0\pi, \frac{2}{3}\pi, \frac{4}{3}\pi, 2\pi \\ x = \frac{\pi}{3}, \pi, \frac{5\pi}{3} \end{cases}}$

$2x = \arcsin(0)$

$\begin{cases} 2x = \sin^{-1}(0) + 2\pi n \\ 2x = \pi - \sin^{-1}0 + 2\pi n \end{cases}$

$\begin{cases} 2x = 0 + 2\pi n \\ 2x = \pi + 2\pi n \end{cases} \Rightarrow \begin{cases} x = 0 + \pi n \\ x = \frac{\pi}{2} + \pi n \end{cases}$

$\boxed{\begin{cases} x = 0, \pi, 2\pi \\ x = \frac{\pi}{2}, \frac{3\pi}{2} \end{cases}}$