Section 5-6 Double and Half Argument Properties

**Prove Double Argument Properties:**

$$\sin(\left(2A\right)=\sin((A+A)))$$

 = sinAcosA + cosAsinA

 =2sinAcosA

sin(2A) = 2sinAcosA

cos(2A) = cos(A + A)

 = cosAcosA – sinAsinA

 =$cos^{2}A-sin^{2}A $

cos(2A) = $cos^{2}A-sin^{2}A$

OR

cos(2A) = $cos^{2}A-sin^{2}A$

$$ =cos^{2}A-\left(1-cos^{2}A\right)$$

 = $cos^{2}A-1$ + $cos^{2}A$

 = 2$cos^{2}A-1$

cos(2A) = 2$cos^{2}A-1$

OR

cos(2A) = $cos^{2}A-sin^{2}A$

 = (1 $-sin^{2}A$) $-sin^{2}A$

 = 1$-2sin^{2}A$

cos(2A) = 1$-2sin^{2}A$

tan(2A) = tan(A + A)

 = $\frac{tanA+tanA}{1-tanAtanA}$

tan(2A) = $\frac{2tanA}{1-tan^{2}A}$

**Prove Half-Argument Properties:**

Take cos(2A) = 2$cos^{2}A-1$

 2$cos^{2}A$ = 1 + cos(2A)

 $cos^{2}A=\frac{1+cos⁡(2A)}{2}$ The argument A on the left is half the argument 2A on the right.

Substituting B for 2A leads to

 $cos^{2}\frac{B}{2}= \frac{1+cos⁡(B)}{2}$ Now take the square root of both sides

 cos $\frac{B}{2}$ = $\pm \sqrt{\frac{1+cos⁡(B)}{2} }$ + or – is determined by the Quadrant of $\frac{B}{2}$

Next, cos(2A) = 1$-2sin^{2}A$

 $2sin^{2}A=1-cos⁡(2A)$

 $sin^{2}A=\frac{1-cos⁡(2A)}{2}$ Substitute B for 2A.

 $sin^{2}\frac{B}{2} = \frac{1-cos⁡(B)}{2}$ Now take the square root of both sides.

 sin $\frac{B}{2}$ = $\pm \sqrt{\frac{1-cos⁡(B)}{2} }$ + or – is determined by the Quadrant of $\frac{B}{2}$

A little more difficult to prove tan$\frac{B}{2}$ so we shall not get into it right now…

tan $\frac{B}{2}$ =$\pm \sqrt{\frac{1-cosB}{1+cosB} }$ $= \frac{1-cosB}{sinB}= \frac{sinB}{1+cosB}$ + or – is determined by the Quadrant of $\frac{B}{2}$

**Example 1**:

Use the double argument property, cos(2A) = 1$-2sin^{2}A $to express cos 120$°$ in terms of sin 60$°$

**Example 2**:

Suppose that A is an angle between 270$° $and 360$°$ and that cos A = $\frac{5}{13}$

Find the exact value of: cos 2A and sin ½A

**Example 3**:

Suppose that A is an angle between 180$°$ and 270$°$ and that cos A = $\frac{-5}{13}$

Find the exact value of: sin 2A and cos ½A