Solve the equation. (Use trig properties to transform the equation)

1. $\cos^2\theta - \sin^2\theta = -1$ $\quad \theta \in [0,360°]$

Option 1
* what we've done in past, but longer & more complicated

$(1-\sin^2\theta) - \sin^2\theta = -1$

$1 - 2\sin^2\theta = -1$

$0 = 2\sin^2\theta - 2$

$0 = 2(\sin^2\theta - 1)$

$2 = 0$

(Option 2 on back) → new + improved (shorter + easier)

$\sin^2\theta - 1 = 0$

$\sin^2\theta = 1$

$\sin\theta = \pm 1$

$\theta = \sin^{-1}(1) + 360°n$

$\theta = 180 - \sin^{-1}(1) + 360°n$

$90° + 360°n$

$\theta = \sin^{-1}(-1) + 360°n$

$\theta = 180 - \sin^{-1}(-1) + 360°n$

$270° + 360°n$

$90° \& 270°$

2. $\frac{2\tan x}{1-\tan^2 x} = \sqrt{3}$ $\quad x \in [0, 2\pi]$ 6.28

$\tan 2A = \sqrt{3}$

$2A = \tan^{-1}(\sqrt{3}) + \pi n$

$\frac{2A}{2} = \frac{1.047}{2} + \frac{\pi n}{2}$ OR $\frac{2A}{2} = \frac{\frac{\pi}{3}}{2} + \frac{\pi n}{2}$

$A = 0.5236 + \frac{\pi}{2}n$ OR $A = \frac{\pi}{6} + \frac{\pi}{2}n$

$\frac{\pi}{6} + \frac{3\pi}{6}$

0.5236
2.094
3.665
5.236

OR

$\frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$

3. $\sqrt{\frac{1}{2}(1-\cos\theta)} = 1$ $\quad \theta \in [0,720°]$

** Note: $\sin\frac{\theta}{2}$, $\cos\frac{\theta}{2}$ & $\tan\frac{\theta}{2}$ are ALL $\pm$, so we have to consider $\sin\frac{\theta}{2} = -1$ as well.

$\sin\frac{\theta}{2} = 1$

$\frac{\theta}{2} = \sin^{-1}(1) + 360°n$

$\frac{\theta}{2} = 180 - \sin^{-1}(1) + 360°n$ → ends up being same as above

$2 * \frac{\theta}{2} = (90 + 360°n) * 2$

$\theta = 180° + 720°n$

$\theta = 180°$

$\sin\frac{\theta}{2} = -1$

$\frac{\theta}{2} = \sin^{-1}(-1) + 360°n$

$\frac{\theta}{2} = -90 + 360°n$

$\theta = -180° + 720°n$

$\theta = -180°, 540°$

$\frac{\theta}{2} = 180 - \sin^{-1}(-1) + 360°n$ (same)

#1 Option 2

$\cos^2\theta - \sin^2\theta = -1$

$\cos 2\theta = -1$

$2\theta = \pm \cos^{-1}(-1) + 360°n$

$\frac{2\theta}{2} = \frac{\pm 180°}{2} + \frac{360°n}{2}$

$\theta = \pm 90° + 180°n$

$90°, 270°$

$-90, 90°$