Prove Double Argument Properties:

$$\begin{aligned}\sin(2A) &= \sin(A+A)\\ &= \sin A\cos A + \cos A\sin A\\ &= 2\sin A\cos A\end{aligned}$$

$$\boxed{\sin(2A) = 2\sin A\cos A}$$

$$\begin{aligned}\cos(2A) &= \cos(A+A)\\ &= \cos A\cos A - \sin A\sin A\\ &= \cos^2 A - \sin^2 A\end{aligned}$$

$$\boxed{\cos(2A) = \cos^2 A - \sin^2 A}$$

OR

$$\begin{aligned}\cos(2A) &= \cos^2 A - \sin^2 A\\ &= \cos^2 A - (1-\cos^2 A)\\ &= \cos^2 A - 1 + \cos^2 A\\ &= 2\cos^2 A - 1\end{aligned}$$

$$\boxed{\cos(2A) = 2\cos^2 A - 1}$$

OR

$$\begin{aligned}\cos(2A) &= \cos^2 A - \sin^2 A\\ &= (1-\sin^2 A) - \sin^2 A\\ &= 1 - 2\sin^2 A\end{aligned}$$

$$\boxed{\cos(2A) = 1 - 2\sin^2 A}$$

$$\begin{aligned}\tan(2A) &= \tan(A+A)\\ &= \frac{\tan A + \tan A}{1 - \tan A\tan A}\end{aligned}$$

$$\boxed{\tan(2A) = \frac{2\tan A}{1-\tan^2 A}}$$

Prove Half-Argument Properties:

Take $\cos(2A) = 2cos^2A - 1$

$2cos^2A = 1 + \cos(2A)$

$cos^2A = \frac{1+\cos(2A)}{2}$ The argument A on the left is half the argument 2A on the right.

Substituting B for 2A leads to

$cos^2\frac{B}{2} = \frac{1+\cos(B)}{2}$ Now take the square root of both sides

$$\boxed{\cos\frac{B}{2} = \pm\sqrt{\frac{1+\cos(B)}{2}}}$$

+ or – is determined by the Quadrant of $\frac{B}{2}$

Next, $\cos(2A) = 1-2sin^2A$

$2\sin^2A = 1 - \cos(2A)$

$\sin^2A = \frac{1-\cos(2A)}{2}$ Substitute B for 2A.

$\sin^2\frac{B}{2} = \frac{1-\cos(B)}{2}$ Now take the square root of both sides.

$$\boxed{\sin\frac{B}{2} = \pm\sqrt{\frac{1-\cos(B)}{2}}}$$

+ or – is determined by the Quadrant of $\frac{B}{2}$

A little more difficult to prove $\tan\frac{B}{2}$ so we shall not get into it right now…

$$\boxed{\tan\frac{B}{2} = \pm\sqrt{\frac{1-cosB}{1+cosB}} = \frac{1-cosB}{sinB} = \frac{sinB}{1+cosB}}$$

+ or – is determined by the Quadrant of $\frac{B}{2}$

Example 1:

Use the double argument property, $\cos(2A) = 1-2\sin^2A$ to express cos 120° in terms of sin 60°

$2A = 120°$

$A = 60°$

$\cos 120° = 1 - 2\sin^2 A$

$\cos(2\cdot 60°) = 1 - 2\sin^2 A$

$$\boxed{= 1 - 2\sin^2 60°}$$

Example 2:

Suppose that A is an angle between 270° and 360° and that $\cos A = \frac{5}{13}$
Find the exact value of: cos 2A and sin ½A

* because given cos A, we choose:

$$\cos 2A = 2\cos^2 A - 1$$
$$= 2(\cos A)^2 - 1$$
$$= 2\left(\frac{5}{13}\right)^2 - 1$$
$$= 2\left(\frac{25}{169}\right) - 1$$
$$\boxed{\cos 2A = \frac{-119}{169}}$$

$$\sin \tfrac{1}{2}A = \pm\sqrt{\frac{1-\cos A}{2}}$$
$$= \pm\sqrt{\frac{1-\frac{5}{13}}{2}}$$
$$= \pm\sqrt{\frac{\frac{8}{13}}{2}}$$

$\frac{8}{13} \div 2 =$

$\frac{8}{13} * \frac{1}{2}$

$$= \pm\sqrt{\frac{4}{13}}$$
$$= \pm\frac{2}{\sqrt{13}} \text{ or } \pm^{*}\frac{2\sqrt{13}}{13}$$

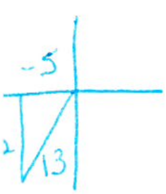
*** pos or neg determined by where $\frac{1}{2}\theta$ is.
if A is b/w 270 & 360 → $\frac{1}{2}$A is b/w 135 & 180 (Quad 2)
AND: sine is pos. in the 2nd quad

$$\boxed{\text{so } = \frac{2}{\sqrt{13}} \text{ or } \frac{2\sqrt{13}}{13}}$$

Example 3:

Suppose that A is an angle between 180° and 270° and that $\cos A = \frac{-5}{13}$
Find the exact value of: sin 2A and cos ½A

$$\sin 2A = 2\sin A\cos A$$
$$= 2\left(\frac{12}{13}\right)\left(\frac{-5}{13}\right)$$
$$\boxed{= \frac{-120}{169}}$$



$$\cos \tfrac{1}{2}A =$$
$$\pm\sqrt{\frac{1+\cos B}{2}}$$
$$= \pm\sqrt{\frac{1-\frac{5}{13}}{2}}$$
$$= \pm\sqrt{\frac{\frac{8}{13}}{2}}$$
$$= \pm\sqrt{\frac{4}{13}}$$
$$= \pm\frac{2}{\sqrt{13}} \text{ or } \pm\frac{2\sqrt{13}}{13}$$

A b/w 180° & 270°
$\frac{1}{2}$A b/w 90° & 135°
so Quad II (cosine neg in Quad. II)

$$= \boxed{-\frac{2}{\sqrt{13}} \text{ or } -\frac{2\sqrt{13}}{13}}$$