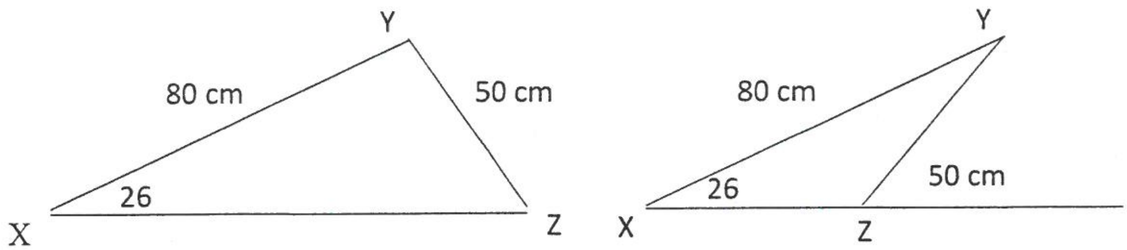
Section 6-5 The Ambiguous Case **ASS**



1. In ΔXYZ above, find all possible side lengths of y.

Law of Sines $\frac{\sin 26°}{50} = \frac{\sin Z}{80}$

$Z = \arcsin(0.70139)$
$Z = 44.5°$ or $180° - 44.5° = 135.5°$

* Always check angle-side relationships (is smaller ∠ across from smallest side, etc.)

Case 1

$\angle X = 26°$
$x = 50$ cm
$z = 80$ cm
$\angle Z = 44.5°$
$\angle Y = 109.5°$
$y = 107.5$ cm

* find 3rd ∠ by 180 − (other 2)

Case 2

$\angle X = 26°$
$x = 50$ cm
$z = 80$ cm
$\angle Z = 135.5°$
$\angle Y = 18.5°$
$y = 36.2$

Side y:

$\frac{\sin 26}{50} = \frac{\sin 109.5}{y}$

$y = 107.5$ cm

OR

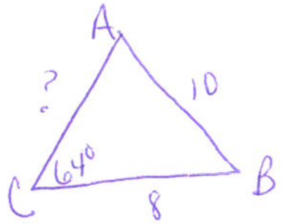
$\frac{\sin 26}{50} = \frac{\sin 18.5}{y}$

$y = 36.2$ cm

*** Not all cases of ASS have 2 solutions though! Depends on side lengths and if given angle is obtuse or acute. Just check angle measurements after arcsin(X) to see if they indeed form a triangle.**

2. $C = 64°$, $c = 10$ft, $a = 8$ft. Find all missing parts of triangle.

This time, let's use Law of Cosines (which will help us figure out # of solutions right away)



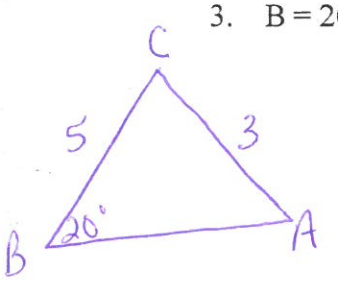
$10^2 = 8^2 + b^2 - 2(8)(b)(\cos 64)$

$36 = b^2 - 7.01b$

$0 = b^2 - 7.01b + 36$

$\left.\begin{array}{l} 3.505 + 4.869i \\ 3.505 - 4.869i \end{array}\right\}$ because answers are imaginary, this triangle has $\boxed{\text{NO SOLUTIONS}}$

3. B = 20°, a = 5, b = 3. Find all missing parts of triangle.



$3^2 = 5^2 + c^2 - 2(5)(c)(\cos 20)$

$-16 = c^2 - 9.4c$

$0 = c^2 - 9.4c + 16$

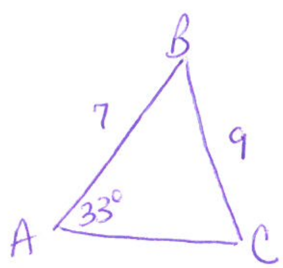
$\boxed{c_1 = 7.17 \text{ or } c_2 = 2.23}$ * so this Δ has 2 sol.

$\frac{\sin 20}{3} = \frac{\sin C}{7.17}$ $\frac{\sin 20}{3} = \frac{\sin C}{2.23}$

$C_1 = 54.8°$ $C_2 = 14.7°$

$A_1 = 105.2°$ $A_2 = 145.3°$

4. A = 33°, a = 9, c = 7. Find all missing parts of triangle.



$9^2 = 7^2 + b^2 - 2(7)(b)(\cos 33°)$

$32 = b^2 - 11.74b$

$0 = b^2 - 11.74b - 32$

$\boxed{b = 14.02}$ or $b = -2.28$

* Since one answer is negative, it can't be the side length of a triangle, so there's only 1 sol. for this.

$\frac{\sin 33}{9} = \frac{\sin B}{14.02}$

$B = 58°$

$\boxed{\text{or } B = 122°}$

(Since this is an ambiguous case & using sin⁻¹, we have to consider 180 − sin⁻¹, then choose the ∠ that makes the ∠/side relationships true)

* Since b is the largest side, ∠B needs to be the biggest. (If use 58°, ∠C would be 89° and the largest, so need to use 122°)