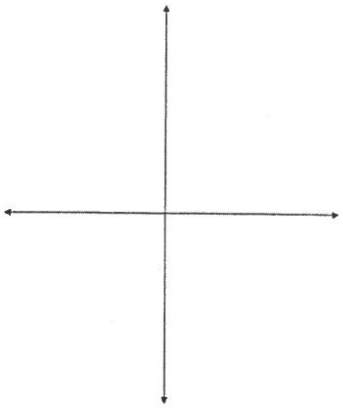
Section 6-6 Vector Addition Continued

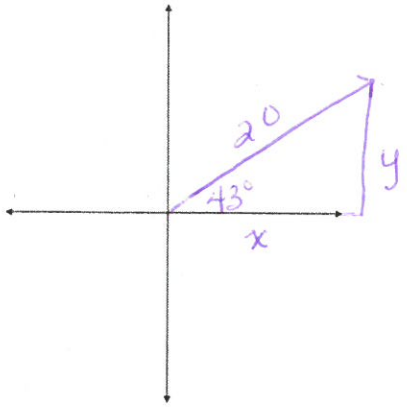
We write vectors with their horizontal and vertical components.

If $\vec{v}$ is a vector in the direction θ in standard position, then
$\vec{v} = x\vec{i} + y\vec{j}$
where $x = |\vec{v}|\cos\theta$ and $y = |\vec{v}|\sin\theta$



Resolve a vector into horizontal and vertical components.

1. Vector $\vec{a}$ has magnitude of 20 and direction of 43° from the horizontal.



$\vec{i}$ = horiz. unit vector
$\vec{j}$ = vert. unit vector

① Draw vector & create rt. Δ
② find x & y using trig.

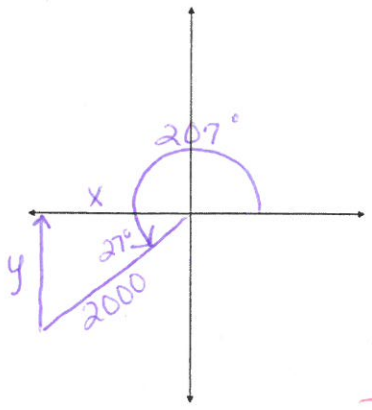
$\sin 43 = \frac{y}{20}$ $\cos 43 = \frac{x}{20}$

$\therefore y = 20(\sin 43)$ $x = 20(\cos 43)$

* pay attention to their direction so you are certain the signs are correct (especially if you are using reference angles.)

$\vec{a} = 14.63\vec{i} + 13.64\vec{j}$

2. Vector $\vec{a}$ has magnitude of 2000 and direction of 207° from the horizontal.



Option 1: ref. θ

$x = 2000\cos 27$
$y = 2000\sin 27$
$x = 1782.01$
$y = 907.98$

* look at direction x & y are going. Both need to be negative

$\vec{a} = -1782\vec{i} - 908\vec{j}$

Option 2: actual θ

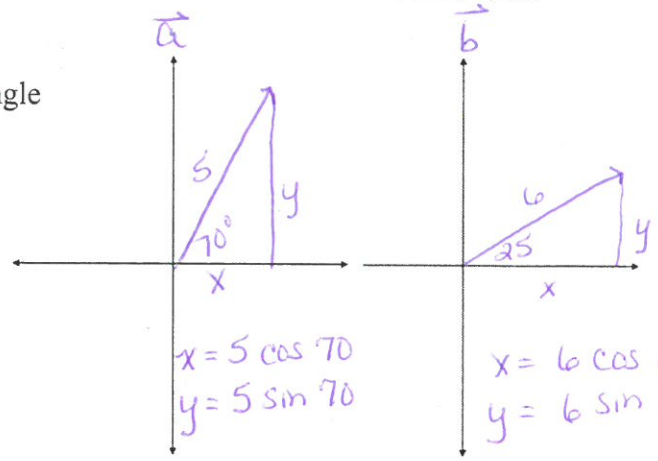
$x = 2000\cos 207$
$y = 2000\sin 207$
$x = -1782.01$
$y = -907.98$

* because using actual θ, signs are already in place

$\vec{a} = -1782\vec{i} - 908\vec{j}$

3. Vector $\vec{a}$ is 5 at 70°, and vector $\vec{b}$ is 6 at 25°. Find the resultant vector, $\vec{r}$, as:

   a. The sum of two components
   b. A magnitude and a direction angle



(1) Break each vector into its components →

(2) Write each vector in terms of its components

$x = 5\cos 70$
$y = 5\sin 70$

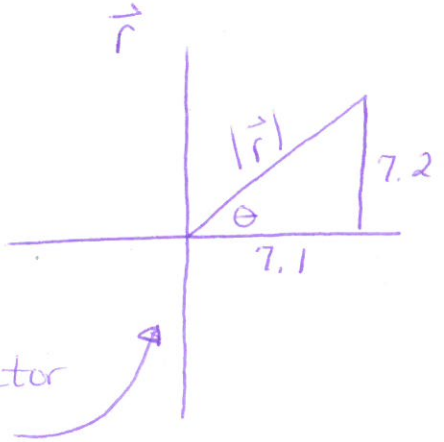
$x = 6\cos 25$
$y = 6\sin 25$

$\vec{a} = 1.7\vec{i} + 4.7\vec{j}$

$\vec{b} = 5.4\vec{i} + 2.5\vec{j}$

a.) $\vec{r} = 7.1\vec{i} + 7.2\vec{j}$

(3) add two vectors

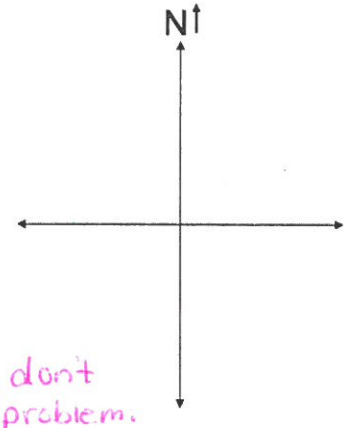


b.
$|\vec{r}| = 10.11$
$\theta = 45.4°$

(4) Draw the resultant vector using the components

(5) Find the magnitude (hypotenuse) and direction (θ) using trigonometry.

A **bearing** is an angle measured clockwise from the North



Example: Bearing of 250°

Example: Bearing of 70°

** Bearing is just a different way to measure the angle. If they confuse you, don't work with them -- just consider @ the end of problem.

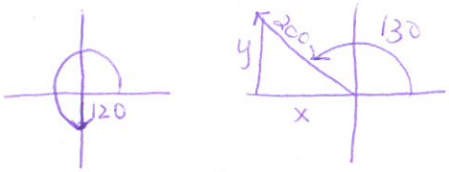
4. A ship moves west (bearing of 270°) for 120 miles and then turns and moves on a bearing of 130° for another 200 miles.
   a) How far is the ship from its starting point?
   b) What is the ship's bearing from its starting point?

* 2 ways to solve:

a.) using components

b.) using bearings (drawings must be pretty good) ☺

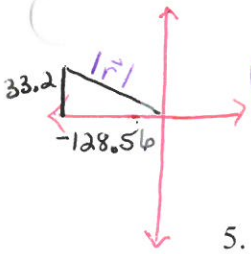
Components:



$0\vec{i} - 120\vec{j}$

$-128.56\vec{i} + 153.2\vec{j}$

$\vec{r} = -128.56\vec{i} + 33.2\vec{j}$



① Find comp. of each vector

② Add together to get resultant

③ Graph $\vec{r}$ on new axes

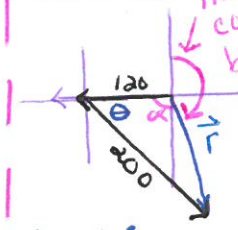
④ Find magnitude (hypotenuse) and θ (use tan⁻¹)

ⓐ $|\vec{r}| = 132.78$ miles

$\theta = -14.48$ (since this would be in 4th, we need to add 180° to get in 2nd)

ⓑ $\theta = 165.5°$

Bearings:



$\theta = 40°$ (130° - 90°)

$|\vec{r}|^2 = 120^2 + 200^2 - 2(120)(200)(\cos 40)$

ⓐ $|\vec{r}| = 132.78$ miles

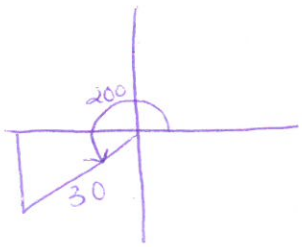
$\frac{\sin 40}{132.78} = \frac{\sin \alpha}{200}$

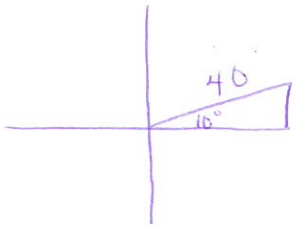
$\alpha = 75.5°$ OR $104.5°$ *

ⓑ $270 - 104.5 = 165.5°$

* must check 3rd ∠ and make sure ∠/side relationships are correct

5. A plane flies 30 miles on a bearing of 200° and then turns and flies 40 miles on a bearing of 10°. Find the resultant displacement vector as a distance and a bearing.





$x = 30\cos 200°$

$y = 30\sin 200°$

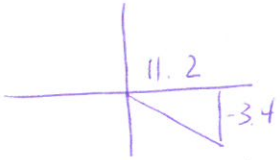
$-28.2\vec{i} - 10.3\vec{j}$

$x = 40\cos 10°$

$y = 40\sin 10°$

$39.4\vec{i} + 6.9\vec{j}$

$\vec{r} = 11.2\vec{i} - 3.4\vec{j}$



$\tan^{-1}\left(\frac{-3.4}{11.2}\right) = -16.89$ -- but we want same angle w/ pos. name, so add 360°

distance

bearing

11.7 miles @ 343.1°