Section 7-4 Logarithms

We know that $2^2 = 4$ and $2^3 = 8$. However, for what value of x does $2^x = 6$?

To find the exact value, mathematicians invented ***logarithms***.

Let b and x be positive numbers, $b \neq 1$. The logarithm of x with base b is

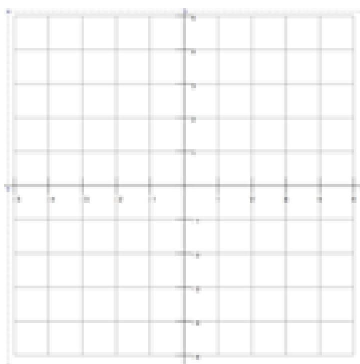
$$\log_b x = y \quad \text{if and only if} \quad b^y = x$$

It is read as "log base b of x".

Logarithms and Exponential Functions are inverses of each other

$y = 2^x$ its inverse $x = 2^y$ or $\log_2 x = y$

x	y
-2	¼
-1	½
0	1
1	2
2	4



x	y
¼	-2
½	-1
1	0
2	1
4	2

Rewrite as an exponential function.

1. $\log_3 9 = 2$

2. $\log_5 \frac{1}{25} = -2$

Rewrite an a logarithm.

3. $4^3 = 64$

4. $10^4 = 10{,}000$

The log with base 10 is called the **common logarithm**. It is written $\log_{10} x$ or $\log x$.
The log with base e =2.7182…is called the **natural logarithm.**
It can be written $\log_e x$ but is more often referred to as

Let b, u, and v be positive numbers such that $b \neq 1$.

Product Property $\log_b uv = \log_b u + \log_b v$

Example: $\log_5 21 = \log_5 3 + \log_5 7$

Quotient Property $\log_b \frac{u}{v} = \log_b u - \log_b v$

Example: $\log_5 \frac{3}{7} = \log_5 3 - \log_5 7$

Power Property $\log_b u^n = n \log_b u$

Example: $\log_5 49 = \log_5 7^2 = 2\log_5 7$

Demonstrate numerically the property of logarithms.

5. $\ln(7 \cdot 8) = \ln 7 + \ln 8$

Fill in the blank.

6. log 5 + log 8 = log ______

7. ln 4 – ln 20 = ln ______

8. log 49 = ___ log 7

9. log 100 = _____

CHANGE-OF-BASE formula

$$\log_c u = \frac{\log u}{\log c} \quad \text{or} \quad \log_c u = \frac{\ln u}{\ln c}$$

$$\log_3 7 = \frac{\log 7}{\log 3}$$

10. $\log_2 6$

11. $\log_5 129$

12. $\log_{1/2} 7$

Solve.

13. $4^x = 15$

14. $3^{4x} = 27^{x+1}$

15. $\log_5(x+6) + \log_5(x+2) = 1$

16. $\log_2(2x-1) - \log_2(x+2) = -1$

17. $e^{2x} - 3e^x + 2 = 0$