Section 12-2 Classifying Conic Sections and Transforming Equations

Without graphing, tell whether the graph will be a circle, an ellipse, a parabola or a hyperbola.

1. $\frac{(x+4)^2}{4} + \frac{(y-1)^2}{9} = 1$

ellipse → both quad terms (+) dilations diff

2. $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

ellipse → (see # 1)

3. $\frac{(x+2)^2}{25} - (y-1)^2 = 1$

hyperbola → one quad term (-) the other quad term (+)

4. $\left(\frac{x+3}{4}\right)^2 + \left(\frac{y-1}{4}\right)^2 = 1$

circle → both quad terms (+) dilations same

5. $y = (x-5)^2 - 4$

parabola → only one quad term

6. $-\left(\frac{x-3}{2}\right)^2 + \left(\frac{y+1}{2}\right)^2 = 1$

hyperbola → (see #3)

General Equation of a Conic	$Ax^2 + \overset{*}{B}xy + Cy^2 + Dx + Ey + F = 0$

* will only be an 'xy term' if the conic is rotated

Take equation of #1 $\frac{(x+4)^2}{4} + \frac{(y-1)^2}{9} = 1$ and multiply by 36

$9(x+4)^2 + 4(y-1)^2 = 36$
$9(x^2 + 8x + 16) + 4(y^2 - 2y + 1) = 36$
$9x^2 + 72x + 144 + 4y^2 - 8y + 4 = 36$
$9x^2 + 4y^2 + 72x - 8y + 112 = 0$ *notice no xy term

Transform into standard form (normal looking form).

7. $x^2 + y^2 + 6x - 2y - 6 = 0$

* We know it's a circle because both quad terms (+) & dilations (coeff.) are the same

① $(x^2+6x)+(y^2-2y) = 6$

② $(x^2+6x+9)+(y^2-2y+1) = 6+9+1$

③ $(x+3)^2 + (y-1)^2 = 16$

④ $\frac{(x+3)^2}{16} + \frac{(y-1)^2}{16} = 1$

OR

$\left(\frac{x+3}{4}\right)^2 + \left(\frac{y-1}{4}\right)^2 = 1$

1) group like variables & put constant on other side

2.) complete the square for x and y, then add those values to the right side as well

3.) write left side in factored form & simplify right side

④ set = to 1 and reduce/simplify if necessary

Transform.

8. $x^2 - 25y^2 + 4x + 50y - 46 = 0$

We know it's a hyperbola because one quad term (+) and one quad term (-)

$(x^2+4x) + (-25y^2+50y) = 46$

* this time must factor out numerical GCF before we can complete the square

$(x^2+4x) - 25(y^2+2y) = 46$

$(x^2+4x+4) - 25(y^2+2y+1) = 46 + 4 + 1(-25)$

*be careful when there's a numerical GCF -- must include it when adding to right side

$(x+2)^2 - 25(y+1)^2 = 25$

$\frac{(x+2)^2}{25} - \frac{(y+1)^2}{1} = 1$

OR $\left(\frac{x+2}{5}\right)^2 - \left(\frac{y+1}{1}\right)^2 = 1$

9. $-x^2 + 10x + y - 21 = 0$

* We know it's a parabola because there's only 1 var. squared

$y - 21 = x^2 - 10x$

$y - 21 + 25 = x^2 - 10x + 25$

* only need to complete square once.

$y + 4 = (x-5)^2$

$y = (x-5)^2 - 4$