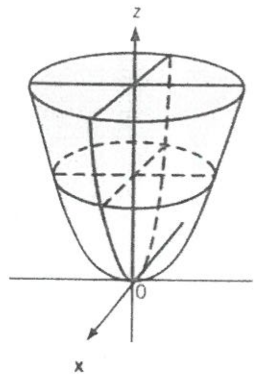
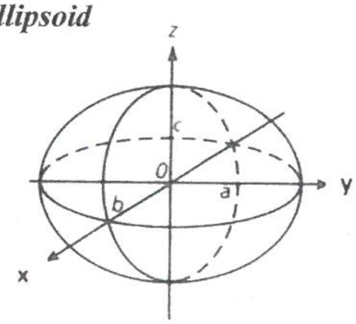
Section 12-3 Quadric Surfaces

A 3-dimensional generated by rotating a conic section around an axis is a **Quadric Surface**.

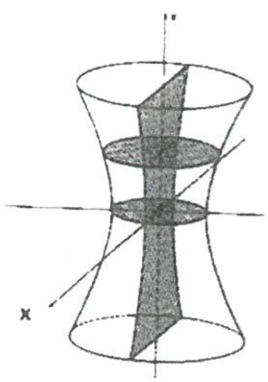
Paraboloid



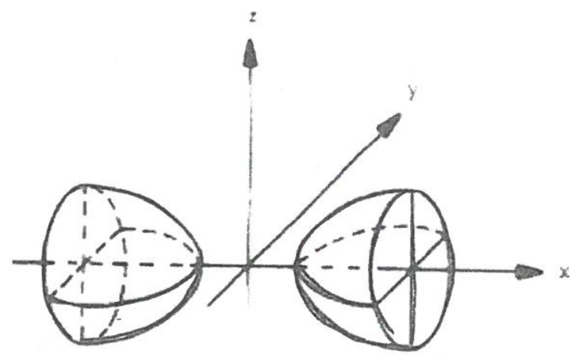
Ellipsoid



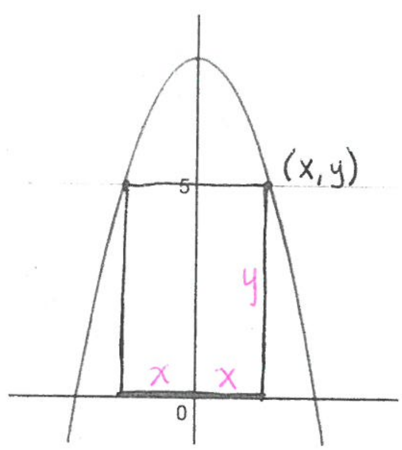
Hyperboloid of one sheet



Hyperboloid of two sheets



1. Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 8 - x^2$



$A = b \cdot h$

$A = 2x \cdot y$

* Since (x, y) is on both the rectangle AND parabola, we can substitute.

$A = 2x(8 - x^2)$ → put in calc & find max

y = max area $\quad x = \frac{1}{2}$ base

max: $(1.63, 17.42)$

↓ $\frac{1}{2}$ base ↓ max area

Dimensions of rectangle:
base = 3.26 un
height = 5.34 un

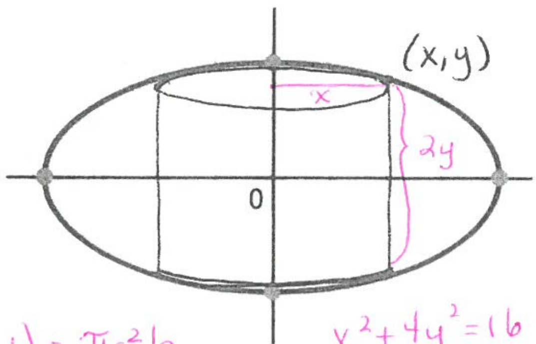
height $= 8 - x^2$
$= 5.34$

OR

$A = b \cdot h$

$17.42 = 3.26\,h \quad h = 5.34$

2. An ellipsoid is formed by rotating ellipse $x^2 + 4y^2 = 16$ about the y-axis. A cylinder is inscribed in the ellipsoid with its axis along the y-axis and its two bases touching the ellipsoid. Find the radius and altitude of the cylinder with maximum volume.



Radius = 3.27

Altitude = 2.30

Max volume = 77.39

$V(\text{cyl}) = \pi r^2 h$

$V = \pi x^2 (2y)$

$V = \pi x^2 \left[2\left(\sqrt{\frac{(16-x^2)}{4}}\right)\right]$

$x^2 + 4y^2 = 16$

$4y^2 = 16 - x^2$

$y = \sqrt{\frac{16-x^2}{4}}$

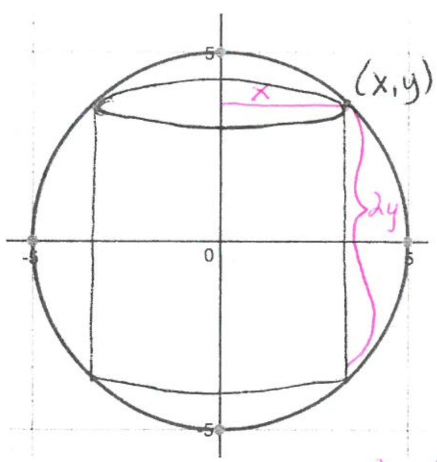
$V = \pi r^2 h$

$77.39 = \pi (3.27)^2 h$

$h = 2.30$

max: $(\underset{\text{radius}}{3.27}, \underset{\text{volume}}{77.39})$

3. A circle $x^2 + y^2 = 25$ is rotated about the y-axis to form a sphere. A cylinder is inscribed in the sphere, with its axis along the y-axis. Write an equation expressing the volume of the cylinder in terms of a sample point (x, y) in the first quadrant.
Find the value of x that gives the maximum volume.



$x = 4.08$

$r = 4.08$

$V = 302.30$

$h = 5.78$

$V = \pi r^2 h$

$302.30 = \pi (4.08)^2 h$

$V = \pi r^2 h$

$V = \pi x^2 (2y)$

$V = \pi x^2 \left[2\left(\sqrt{25 - x^2}\right)\right]$

$x^2 + y^2 = 25$

$y = \sqrt{25 - x^2}$

max: $(\underset{r}{4.08}, \underset{V}{302.30})$