Notes 1.2 Properties of Derivatives

Product Rule

$(fg)' = fg' + gf'$

Differentiate each of the following functions.

1. $y = (x^2 + 1)\sqrt{x}$

$f = (x^2+1) \quad f' = 2x$

$g = \sqrt{x} \quad g' = \frac{1}{2}x^{-\frac{1}{2}}$

$x^{\frac{1}{2}}$

$\boxed{(fg)' = (x^2+1)(\frac{1}{2}x^{-\frac{1}{2}}) + (\sqrt{x})(2x)}$

OR $\boxed{\frac{5}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{-\frac{1}{2}}}$

2. $y = 7x\sqrt{x}$

$f = 7x \quad f' = 7$

$g = \sqrt{x} \quad g' = \frac{1}{2}x^{-\frac{1}{2}}$

$x^{\frac{1}{2}}$

$\boxed{(fg)' = (7x)(\frac{1}{2}x^{-\frac{1}{2}}) + (\sqrt{x})(7)}$

OR $\boxed{\frac{21}{2}x^{\frac{1}{2}}}$

Quotient Rule

$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$

"low d-high, high d-low, square the bottom & away you go"

3. $y = \frac{1+x^2}{1-x^2}$

$f = 1+x^2 \quad f' = 2x$

$g = 1-x^2 \quad g' = -2x$

$\frac{(1-x^2)(2x) - (1+x^2)(-2x)}{(1-x^2)^2}$

$\frac{2x - 2x^3 + 2x + 2x^3}{(1-x^2)^2} = \boxed{\frac{4x}{(1-x^2)^2}}$

4. $y = \frac{3x-5}{2x}$

$f = 3x-5 \quad f' = 3$

$g = 2x \quad g' = 2$

$\frac{2x(3) - (3x-5)(2)}{(2x)^2}$

$\frac{6x - 6x + 10}{4x^2} = \frac{10}{4x^2}$

$\boxed{= \frac{5}{2x^2}}$

Reciprocal Rule
$\left(\frac{1}{g}\right)' = \frac{-g'}{g^2}$

** Keep in mind reciprocals could be written by moving denom. to the numerator.

5. $y = \frac{1}{4x+7}$

$g = 4x+7$

$g' = 4$

$$\boxed{\frac{-(4)}{(4x+7)^2}}$$

OR

$y = (4x+7)^{-1}$

$y' = -1(4x+7)^{-2} * 4$

$y' = -4(4x+7)^{-2}$

or

$\frac{-4}{(4x+7)^2}$

* this may require chain rule (which we don't know yet)

6. $y = \left(\frac{1}{\sqrt{x}}\right)^3$

$y = \frac{1}{(\sqrt{x})^3} = \frac{1}{(x^{\frac{1}{2}})^3} = \frac{1}{x^{\frac{3}{2}}}$

$g = x^{\frac{3}{2}}$ $\quad g' = \frac{3}{2}x^{\frac{1}{2}}$

$$\frac{-\frac{3}{2}x^{\frac{1}{2}}}{(x^{\frac{3}{2}})^2} = \boxed{\frac{-\frac{3}{2}x^{\frac{1}{2}}}{x^3}}$$

$\boxed{\text{OR } -\frac{3}{2}x^{-\frac{5}{2}}}$

OR $y = (\sqrt{x})^{-3}$

$y = (x^{\frac{1}{2}})^{-3}$

$y = x^{-\frac{3}{2}}$

$y' = -\frac{3}{2}x^{-\frac{5}{2}}$