Notes 1.3 **Derivative Notation and the Chain Rule**

There are many different ways to write notation for a derivative:

a) Let $f(x) = 3x^2$ $\quad f'(x) = 6x$

b) We can also say $\frac{d}{dx}(3x^2) = 6x$

$\frac{d}{dx}$ means the function is going to be differentiated with respect to the variable x

c) Let $y = 3x^2$ $\quad y' = 6x$

d) We can say, find $\frac{dy}{dx}$ of $y = 3x^2$

This means to find the derivative of y with respect to x

e) Let y = 3t so $y' = 3$ or $\frac{d}{dt}(3t) = 3$

f) Let $u = x^3$ Find $\frac{du}{dx}$ Answer: $u' = 3x^2$ or $\frac{d}{dx}(x^3) = 3x^2$

Let $y = x^3 - 4x^2 + 5x + 1$

1. Find $\frac{dy}{dx}$ $\quad y' = 3x^2 - 8x + 5$

2. Find the second derivative, called y'' $\quad y'' = 6x - 8$

3. Find the third derivative. $\quad y''' = 6$

4. Find the fourth derivative. $\quad y'''' = 0$

Let $f(x) = -2x^4 + 3x^2$

5. Find $f''(x)$ $\quad f' = -8x^3 + 6x$ $\quad \boxed{f'' = -24x^2 + 6}$

6. Differentiate $\frac{d}{dx}(x + 1)^3$ $\quad \boxed{3(x+1)^2}$

The Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

* Find the derivative of the function as a whole, then mult. by the derivative of the part inside the ( ) or under the $\sqrt{\ }$.

Differentiate.

7. $y = (x^2 - 5)^3$

$y' = 3(x^2-5)^2 * 2x$

$\boxed{y' = 6x(x^2-5)^2}$

8. $y = \sqrt{1 + x^3} = (1+x^3)^{\frac{1}{2}}$

$y' = \frac{1}{2}(1+x^3)^{-\frac{1}{2}} * 3x^2$

$\boxed{y' = \frac{3}{2}x^2(1+x^3)^{-\frac{1}{2}}}$

9. $y = x^2\sqrt{1 - x}$

$f = x^2 \quad f' = 2x$

$g = \sqrt{1-x} = (1-x)^{\frac{1}{2}} \quad g' = \frac{1}{2}(1-x)^{-\frac{1}{2}} * -1$

$g' = -\frac{1}{2}(1-x)^{-\frac{1}{2}}$

$\boxed{(x^2)\left[-\frac{1}{2}(1-x)^{-\frac{1}{2}}\right] + (\sqrt{1-x})(2x)}$

OR

$\boxed{\frac{4x-5x^2}{2\sqrt{1-x}}}$

10. $y = \frac{x^2}{\sqrt{1-x}}$

$f = x^2 \quad f' = 2x$

$g = \sqrt{1-x}$

$g = (1-x)^{\frac{1}{2}} \quad g' = \frac{1}{2}(1-x)^{-\frac{1}{2}} * -1$

$g' = -\frac{1}{2}(1-x)^{-\frac{1}{2}}$

$\boxed{(\sqrt{1-x})(2x) - (x^2)\left[-\frac{1}{2}(1-x)^{-\frac{1}{2}}\right]}$

OR

$\boxed{\frac{4x-3x^2}{2(1-x)^{3/2}}}$