Notes 1.5 Using the Derivative to help in Graphing Functions

****NO GRAPHING CALCULATOR****

We can find the intervals on which a function is increasing/decreasing and where the extreme point(s) are by analyzing the 1st derivative.

Graph the function by finding: y-intercept, extreme point(s), and intervals where *f* is increasing/decreasing.

1. $f(x) = x^2 - 8x + 1$

* y int: (0,1)

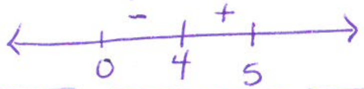
* extreme points are where derivative = 0

$f'(x) = 2x - 8$

$0 = 2x - 8$

$8 = 2x$

$4 = x$

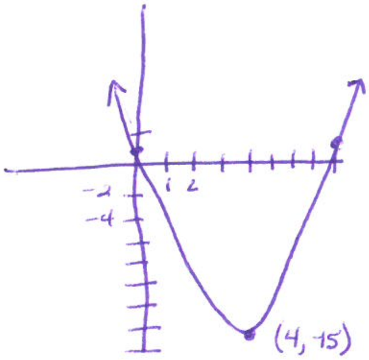


*plug 4 into orig. equation to find

$y \to 4^2 - 8(4) + 1$

$y = -15$

$\boxed{(4,-15)\text{ min}}$



* choose a point to left & right of extreme to determine where graph is inc. or dec. -- plug into derivative. →

$2(0) - 8 = -8$

so dec. to the left

$2(5) - 8 = 2$

so inc to the right

(means 4 is a min)

$(-\infty, 4)$ dec.

$(4, \infty)$ inc.

2. $f(x) = x^3 - 3x^2 - 9x + 22$

* y int: (0,22)

* extreme points

$f'(x) = 3x^2 - 6x - 9$

$0 = 3(x^2 - 2x - 3)$

$0 = 3(x-3)(x+1)$

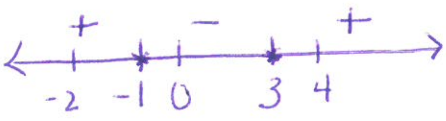
$x = 3 \quad x = -1$

$y = (-1)^3 - 3(-1)^2 - 9(-1) + 22$

$y = 27 \quad \boxed{(-1, 27)\text{ max}}$

$y = (3)^3 - 3(3)^2 - 9(3) + 22$

$y = -5 \quad \boxed{(3,-5)\text{ min}}$



$3(-2)^2 - 6(-2) - 9 = 15$

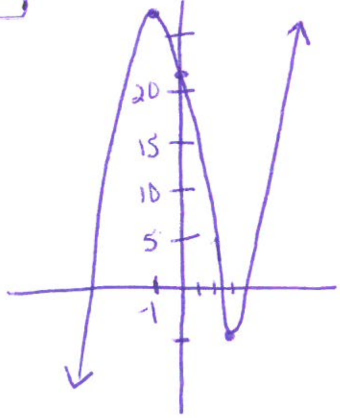
$3(0)^2 - 6(0) - 9 = -9$

$3(4)^2 - 6(4) - 9 = 15$

$(-\infty, -1)$ inc.

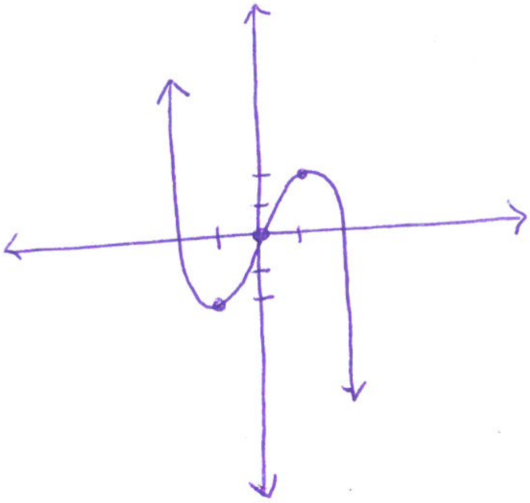
$(-1, 3)$ dec.

$(3, \infty)$ inc.



Graph the function by finding: y-intercept, extreme point(s), and intervals where *f* is increasing/decreasing.

3. $f(x) = -x^3 + 3x$



* y-int: $(0,0)$

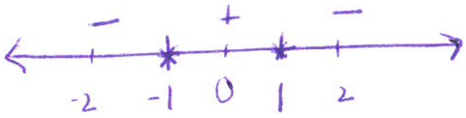
* extreme points

$f'(x) = -3x^2 + 3$

$0 = -3(x^2 - 1)$

$0 = -3(x-1)(x+1)$

$x = 1 \quad x = -1$



$-3(-2)^2 + 3 = -9$

$-3(0)^2 + 3 = 3$

$-3(2)^2 + 3 = -9$

$(-\infty, -1)$ dec

$(-1, 1)$ inc

$(1, \infty)$ dec

$y = -(-1)^3 + 3(-1)$

$y = -2 \quad \boxed{(-1,-2) \text{ min}}$

$y = -(1)^3 + 3(1)$

$y = 2 \quad \boxed{(1,2) \text{ max}}$