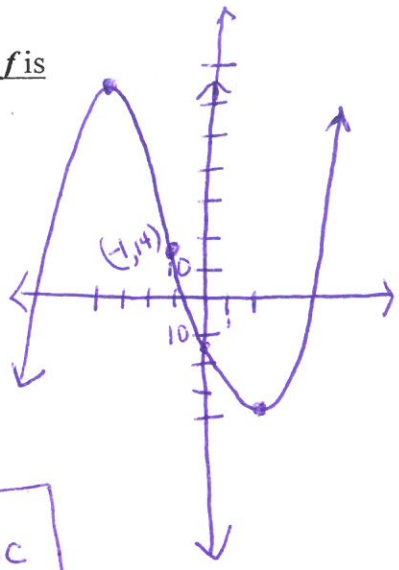
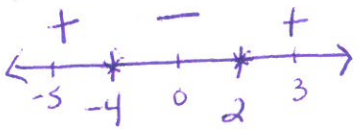
Notes 1.6 The 2nd Derivative and Concavity

From yesterday:
Graph the function by finding: y-intercept, extreme point(s), and intervals where ***f*** is increasing/decreasing.

1. $f(x) = x^3 + 3x^2 - 24x - 12$





* y-int: $(0,-12)$

* extremes:

$f'(x) = 3x^2 + 6x - 24$

$0 = 3x^2 + 6x - 24$

$0 = 3(x^2 + 2x - 8)$

$0 = 3(x+4)(x-2)$

$x = -4 \quad x = 2$

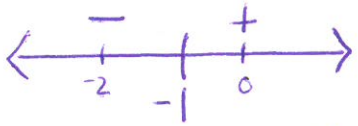
$3(-5)^2 + 6(-5) - 24 = 21$
$3(0)^2 + 6(0) - 24 = -24$
$3(3)^2 + 6(3) - 24 = 21$

$(-\infty, -4) \cup (2, \infty)$ inc
$(-4, 2)$ dec

$y = (-4)^3 + 3(-4)^2 - 24(-4) - 12 = 68$
$y = (2)^3 + 3(2)^2 - 24(2) - 12 = -40$

$(-4, 68)$ max
$(2, -40)$ min

Now find the 2nd Derivative:



$f''(x) = 6x + 6$

$0 = 6x + 6$

$-6 = 6x$

$x = -1$

$6(-2) + 6 = -6$
$6(0) + 6 = 6$

$(-1, 14)$ pt. of inflection

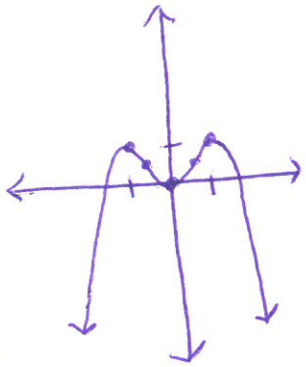
$(-\infty, -1)$ concave down
$(-1, \infty)$ concave up

$y = (-1)^3 + 3(-1)^2 - 24(-1) - 12 = 14$

Points of Inflection are found where the 2nd derivative equals 0. The point of inflection is where the graph changes *concavity*.

Graph the function by finding: y-intercept, extreme point(s), intervals where f is increasing/decreasing, point(s) of inflection, intervals of concave up/down.

2. $f(x) = 2x^2 - x^4$



* y-int: $(0,0)$

* extreme

$f'(x) = 4x - 4x^3$

$0 = 4x(1-x^2)$

$0 = 4x(1-x)(1+x)$

$x = 0 \quad x = 1 \quad x = -1$

$2(0)^2 - (0)^4 = 0$

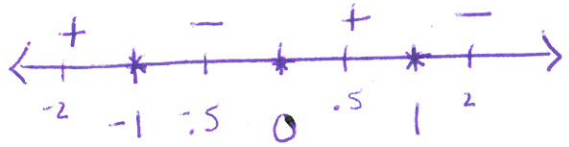
$2(1)^2 - (1)^4 = 1$

$2(-1)^2 - (-1)^4 = 1$

$(0,0)$ min

$(1,1)$ max

$(-1,1)$ max



$4(-2) - 4(-2)^3 = 24$

$4(-.5) - 4(-.5)^3 = -1.5$

$4(.5) - 4(.5)^3 = 1.5$

$4(2) - 4(2)^3 = -24$

$(-\infty, -1)$ and $(0,1)$ inc.

$(-1,0)$ and $(1,\infty)$ dec.

$f''(x) = 4 - 12x^2$

$0 = 4(1 - 3x^2)$

$0 = 1 - 3x^2$

$3x^2 = 1 \quad x^2 = \frac{1}{3}$

$x = \pm 0.577$

$y = 2(0.577)^2 - (0.577)^4$

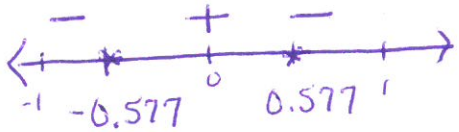
$y = 0.555$

$y = 2(-0.577)^2 - (-0.577)^4$

$y = 0.555$

$(0.577, 0.555)$

$(-0.577, 0.555)$ pts. of inflection



$4 - 12(-1)^2 = -8$

$4 - 12(0)^2 = 4$

$4 - 12(1)^2 = -8$

$(-\infty, -0.577)$ and $(0.577, \infty)$ } concave down

$(-0.577, 0.577)$ concave up