Notes 1.7 **Applications of Derivatives**

1. Find the dimensions of the rectangle having perimeter of 36cm and area as large as possible.

$P = 2\ell + 2w$ $\quad 36 = 2\ell + 2w$

$18 = \ell + w$

$\ell = 18 - w$

$A = \ell \cdot w$

$A = (18-w)w$

$A = 18w - w^2$

* need max, so use derivative to find

$A' = 18 - 2w$

$0 = 18 - 2w$

$2w = 18$

$\boxed{w = 9\text{cm}}$ * need ℓ so go back to original

$\ell = 18 - 9$

$\boxed{\ell = 9\text{cm}}$

2. You decide to fence off a rectangular field that already has a fence on one side, so you only need to fence 3 sides. If you only have 500 feet of fencing available, what dimensions could the field be to enclose the largest area?

$P = 2\ell + w$ $\quad 500 = 2\ell + w$

$w = 500 - 2\ell$

$A = \ell \cdot w$

$A = \ell \cdot (500 - 2\ell)$

$A = 500\ell - 2\ell^2$

$A' = 500 - 4\ell$

$0 = 500 - 4\ell$

$4\ell = 500$

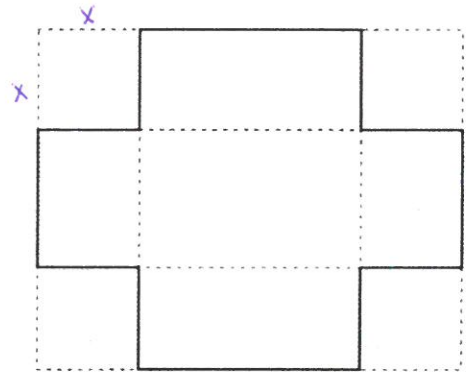
$\boxed{\ell = 125\text{ ft}}$

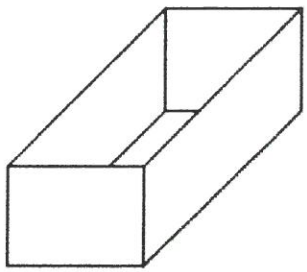
$w = 500 - 2\ell$

$w = 500 - 2(125)$

$\boxed{w = 250\text{ ft}}$

3. A box without a top is made from a 18 inch by 24 inch piece of cardboard by cutting a square from each corner and folding up the sides. How large a square should you cut from each corner so the box has the greatest volume?





$l = 24 - 2x$

$w = 18 - 2x$

$h = x$

$V = l \cdot w \cdot h$

$V = (24-2x)(18-2x)(x)$

$V = (432 - 48x - 36x + 4x^2)x$

$V = 432x - 48x^2 - 36x^2 + 4x^3$

$V = 432x - 84x^2 + 4x^3$

$V' = 432 - 168x + 12x^2$

$0 = 12x^2 - 168x + 432$

$0 = 12(x^2 - 14x + 36)$

36

-14

*not factorable

$x = \frac{14 \pm \sqrt{(14)^2 - 4(1)(36)}}{2(1)}$

$x = \frac{14 \pm \sqrt{52}}{2}$

$x = 10.6$

$l = 2.8$

$w = -3.2$

* doesn't make sense

$x = 3.4 \rightarrow$ so cut

$l = 17.2"$

$w = 11.2"$

$h = 3.4"$

} makes sense

3.4" x 3.4" square