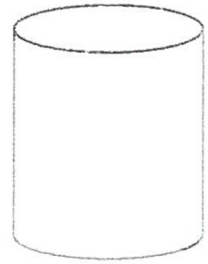
Notes 1.7 continued **Applications of Derivatives**

1. Find the radius and height of the least expensive closed cylinder which has a volume of 100 cubic inches. The materials for making the top cost 2 cents per square inch, the material for bottom costs 5 cents per square inch, and the material for the sides of the can cost 3 cents per square inch.



$V = \pi r^2 h \qquad r = x \quad h = y$

$100 = \pi x^2 y \qquad \frac{100}{\pi x^2} = y$

$L.S.A. = 2\pi x \cdot y \qquad$ Area of top + bottom $= \pi r^2$

cost of L.S.A $= .03(2\pi x y)$

cost of top $= .02(\pi x^2)$

cost of bottom $= .05(\pi x^2)$

Want to minimize cost:

$C = .03\left[2\pi x\left(\frac{100}{\pi x^2}\right)\right] + .02(\pi x^2) + .05(\pi x^2)$

$C = .06\pi x\left(\frac{100}{\pi x^2}\right) + .07\pi x^2$

$C = \frac{6}{x} + .07\pi x^2$

$C = 6x^{-1} + 0.22x^2$

$C' = .44x - 6x^{-2}$

$0 = .44x - 6x^{-2}$

$6x^{-2} = .44x$

$x^2 * \frac{6}{x^2} = .44x * x^2$

$6 = .44x^3$

$13.64 = x^3$

$x = 2.39''$

$y = \frac{100}{\pi(2.39)^2} = 5.57''$

$r = 2.39''$
$h = 5.57''$