

Notes 13.2 Continued

Graphs of Polar Equations

A limacon is a figure with polar equation

$$r = a + b \cos \theta \quad \text{or} \quad r = a + b \sin \theta$$

If $a < b$, then it has an inner loop

$$r = 1 + 2\cos\theta$$

If $a > b$, then it has **no** inner loop

$$r = 3 + 2\cos\theta$$

If $a = b$, then it is heart shaped (**cardioid**)

$$r = 1 - \sin\theta$$

$$r = 2\cos(2\theta) \quad \text{example of a rose}$$

*If number next to θ is even, then $2n$ petals.

$$r = 6\sin(5\theta)$$

If number next to θ is odd, then n petals.

Convert equations from Polar to Cartesian form.

*check by graphing

1. $r = 4$

$$\boxed{r^2 = 16} \rightarrow \text{circle}$$

3. $r = -4\cos\theta$

$$\begin{aligned} r^2 &= -4r\cos\theta \\ x^2 + y^2 &= -4x \\ x^2 + 4x + y^2 &= 0 \rightarrow \text{circle} \end{aligned}$$

5. $r\sin\theta = 3$

$$\boxed{y = 3 \rightarrow \text{line}}$$

2. $r = 6\cos\theta$

$$\begin{aligned} r^2 &= 6r\cos\theta \\ x^2 + y^2 &= 6x \\ x^2 - 6x + y^2 &= 0 \rightarrow \text{circle!} \end{aligned}$$

4. $r = 2\sin\theta$

$$\begin{aligned} r^2 &= 2r\sin\theta \\ x^2 + y^2 &= 2y \\ x^2 + y^2 - 2y &= 0 \rightarrow \text{circle!} \end{aligned}$$

* recall:

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ r^2 &= x^2 + y^2 \end{aligned}$$

7. $r = \frac{9}{5-4\cos\theta} \quad * 5-4\cos\theta$

$$r(5-4\cos\theta) = 9$$

$$5r - 4r\cos\theta = 9$$

$$5\sqrt{x^2 + y^2} - 4x = 9$$

$$(5\sqrt{x^2 + y^2})^2 = (4x + 9)^2$$

$$25(x^2 + y^2) = (4x + 9)(4x + 9)$$

$$25x^2 + 25y^2 = 16x^2 + 72x + 81$$

$$9x^2 - 72x + 25y^2 - 81 = 0$$

ellipse

#8 on back

#8

$$r = \frac{9}{4+5\cos\theta} * 4+5\cos\theta$$

$$r(4+5\cos\theta) = 9$$

$$4r + 5r\cos\theta = 9$$

$$4\sqrt{x^2+y^2} + 5x = 9$$

$$(4\sqrt{x^2+y^2})^2 = (5x+9)^2$$

$$16(x^2+y^2) = (-5x+9)(-5x+9)$$

$$16x^2 + 16y^2 = 25x^2 - 90x + 81$$

$$0 = 9x^2 - 90x - 16y^2 + 81$$

hyperbola