Notes 13.4 Complex Numbers in Polar Form

A complex number: $\mathbf{z = a + b}i$ where a is real part of z, b is imaginary part

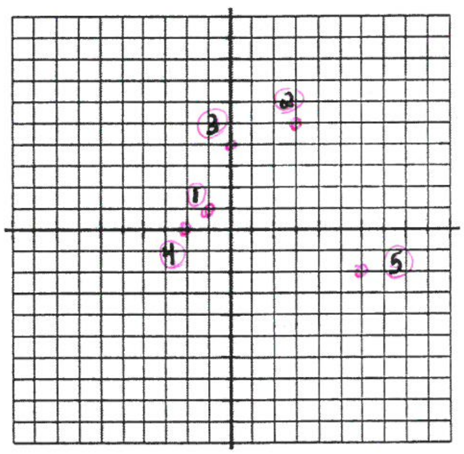
$i = \sqrt{-1}$ imaginary number

$i^2 = -1$ squaring an imaginary removes the radical (so i^2 is real!)

Graph just like graphing (x, y) but there is a real axis and an imaginary axis.

imaginary axis (y)

Real axis (x)



Plot:

1. $z = -1 + i$
2. $z = 3 + 5i$
3. $z = 4i$
4. $z = -2$
5. $z = 6 - 2i$

Review of adding and multiplying complex numbers:

6. $(4 + 3i) - (5 - 2i)$

$4 + 3i - 5 + 2i$

$\boxed{-1 + 5i}$

7. $(4 + 3i)(5 - 2i)$

* remember $i^2 = -1$

$20 - 8i + 15i - 6i^2$

$20 + 7i - 6(-1)$

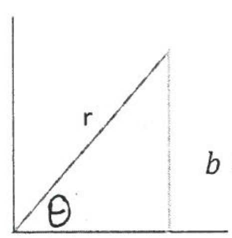
$20 + 7i + 6 \Rightarrow \boxed{26 + 7i}$

$\mathbf{z = a + b}i$

$z = r\cos\theta + r\sin\theta\, i$

$z = r(\cos\theta + i\sin\theta)$

written as "cis θ" pronounced "sis"



$b = r\sin\theta$

$a = r\cos\theta$

any **complex # can be written in Polar Form**

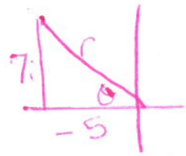
$z = r cis\theta = r(\cos\theta + i\sin\theta)$

r is magnitude (or called modulus)

θ is argument (or angle in degrees or radians)

Write in Polar form. → r cis θ

8. $z = -5 + 7i$

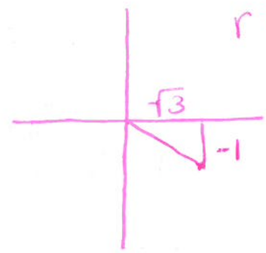


$r = \sqrt{(-5)^2 + (7)^2}$ $r = \sqrt{74}$

$\theta = \tan^{-1}\left(\frac{7}{-5}\right) = -54.5$
$+180$
$125.5°$

$\boxed{\sqrt{74} \text{ cis } 125.5°}$

9. $z = \sqrt{3} - i$

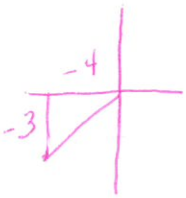


$r = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{4} = 2$

$\theta = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = -30°$

$\boxed{2 \text{ cis } (-30°) \text{ OR } 2 \text{ cis } 330°}$

10. $z = -4 - 3i$



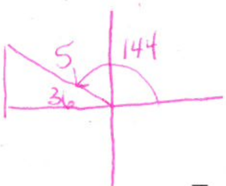
$r = \sqrt{(-4)^2 + (-3)^2} = \sqrt{25} = 5$

$\theta = \tan^{-1}\left(\frac{-3}{-4}\right) = 36.87$
$+180$
216.87

$\boxed{5 \text{ cis } 216.87°}$

Write the complex number in $a + bi$ form.

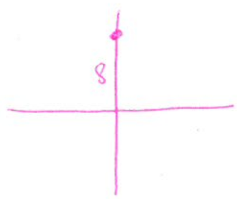
11. 5cis144°



$a(x) = r\cos\theta = -4.05$
$b(y) = r\sin\theta = 2.94$

$\boxed{-4.05 + 2.94i}$

12. 8cis90°



$a(x) = r\cos\theta = 0$
$b(y) = r\sin\theta = 8$

$\boxed{0 + 8i \text{ OR } 8i}$

Evaluate the expressions.

13. (3cis83°)(2cis41°)
multiply magnitudes, add θ

$3 * 2$
$83° + 41°$

$\boxed{6 \text{ cis } 124°}$

14. $\frac{5cis71°}{2cis29°}$
divide magnitudes, subtract θ

$\frac{5}{2}$
$71° - 29°$

$\boxed{\frac{5}{2} \text{ cis } 42°}$

15. $(2cis29°)^5$
raise magnitude to given exponent,
multiply θ by the given exponent

$2^5 \text{ cis } 5 \cdot 29°$

$\boxed{32 \text{ cis } 145°}$

* all angles could be ± 360n, but only listing solutions in 0 ≤ θ < 360° (no coterminals)

16. $(8cis60°)^{\frac{1}{3}}$ * will have 3 answers

$8^{\frac{1}{3}} \text{ cis } \frac{1}{3}(60 + 360°n)$

$2 \text{ cis } 20° + 120°n$

$\boxed{\begin{array}{l} 2 \text{ cis } 20° \\ 2 \text{ cis } 140° \\ 2 \text{ cis } 260° \end{array}}$ when n = 0, n = 1, n = 2

2 cis 380° → don't list it's coterminal to 20°

17. $(81cis64°)^{\frac{1}{4}}$ * 4 answers

$81^{\frac{1}{4}} \text{ cis } \frac{1}{4}(64 + 360°n)$

$3 \text{ cis } 16 + 90°n$

$\boxed{\begin{array}{ll} 3 \text{ cis } 16° & 3 \text{ cis } 196° \\ 3 \text{ cis } 106° & 3 \text{ cis } 286° \end{array}}$

FYI
* sometimes fractional powers written as roots.
i.e #16 $\sqrt[3]{(8 \text{ cis } 60°)}$