Chapter 14 Sequences and Series

Notes 14.2 Arithmetic and Geometric Sequences and Other Sequences

Find the next 3 terms of each sequence.

1. 3, 5, 7, 9, 11, 13 (+2) — A Formula: $t_n = 3 + 2(n-1)$

2. 6, 12, 24, 48, 96, 192 (×2) — G Formula: $t_n = 6 * 2^{(n-1)}$

3. 3, 9, 27, 81, 243, 729 (×3) — G Formula: $t_n = 3 * 3^{(n-1)}$

4. 5, -2, -9, -16, -23, -30 (−7) — A Formula: $t_n = 5 + (-7)(n-1)$

A **sequence** is function whose domain is the set of positive integers. n is the term number and t_n is the term value. (ex #1: the 1st term is 3 so n = 1 and $t_n = 3$)

Arithmetic sequence—each term is formed by *adding* a constant to the previous term. (the constant is called the **common difference** (d)) $t_n = t_1 + d(n-1)$ OR $t_n = t_0 + dn$

Geometric sequence—each term is formed by *multiplying* the previous term by a constant (called the **common ratio** (r)) $t_n = t_1 * r^{(n-1)}$ OR $t_n = t_0 * r^n$

Explicit formulas

Recursive formula: $t_n = t_{n-1} + d$ (Arithmetic) OR $t_n = t_{n-1} * r$ (Geometric)

5. a) Find the 100th term, t_{100}, of the sequence 3, 5, 7, … Arithmetic (+2) $d = 2$ $t_{100} = 3 + 2(100-1)$ $t_{100} = 201$

b) Find the term number, n, that 105 is in the sequence 3, 5, 7, …

$105 = 3 + 2(n-1)$

$102 = 2(n-1)$

$51 = n-1$

$52 = n$

6. a) Find the 100th term, t_{100}, of the sequence 6, 12, 24, … Geometric (×2) $r = 2$ $t_{100} = 6 * 2^{(100-1)}$ $t_{100} = 3.8 * 10^{30}$

b) Find the term number, n, that 786,432 is in the sequence 6, 12, 24, …

$786,432 = 6 * 2^{n-1}$

$131,072 = 2^{n-1}$

$\log 131,072 = (n-1)\log 2$

$\frac{\log 131,072}{\log 2} = n-1$

$17 = n-1$

$18 = n$

7. You have \$40 saved for something. You take on a part-time job that pays \$13 per day. Each day you keep track of how much you have.

Days (n)	\$ or t_n
1	53
2	66
3	79
4	92

+13 +13 +13

a) What kind of sequence is this? Arithmetic

b) How much money would you have after 3 months? Assuming 30 days per month use $n = 90$. $t_{90} = 53 + (90-1)13$ \$1210

c) How long would it take to save \$5000?

$5000 = 53 + 13(n-1)$

$380.54 = n-1$ $\quad n = 381.54 \approx$ 382 days

8. When you leave money in a savings account, the interest is compounded. Let's say you put \$1000 in an account for your baby when it is born and the interest is 6% per year (compounded once a year).

t_0

a) What kind of sequence is this? Write a formula. $t_n = 1000 * 1.06^n$

Geometric

b) Find the first 3 terms.

$t_1 = 1060$ $\quad t_2 = 1123.60$ $\quad t_3 = 1191.02$

c) How much money would there be saved on the 18th birthday?

$t_n = 1000 * 1.06^{18}$ \$2854.34

d) When would that person have \$10,000 saved?

$10,000 = 1000 * 1.06^n$ $\quad 10 = 1.06^n$

$\frac{\log 10}{\log 1.06} = n$ $\quad n = 39.5$ $\approx$ 40 years

9. Type of sequence: Geometric, arithmetic or neither?

n	t_n
1	6
2	12
3	20
4	30
5	42

$\frac{12}{6} = 2$ $\quad \frac{20}{12} \neq 2$ not Geometric

$12-6 = 6$ $\quad 20-12 \neq 6$ not Arithmetic