Notes 14-3 Series and Partial Sums

A **series** is formed by adding the terms of a sequence.

Ex. Sequence 3, 5, 7, 9, … Explicit *Formula*: $t_n = t_0 + dn$, $t_n = t_1 + d(n-1)$ (Arithmetic)

$t_n = t_1 + r^{n-1}$, $t_n = t_0 + r^n$ (Geometric)

Series $3 + 5 + 7 + 9 + \ldots + t_n$

When you sum up part of the series, it is called a **partial sum**, S_n

Ex. The third partial sum is $S_3 = 3 + 5 + 7$
$S_3 = 15$

Find the 100th partial sum, S_{100} of 3 + 5 + 7…

$$S_{100} = \sum_{n=1}^{100} 1 + 2n$$

100 ← ending term number

$1 + 2n$ ← formula for finding t_n (using t_0)

$n=1$ ← beginning term number

Sigma notation

Each series is either arithmetic or geometric. Find the indicated partial sum.

1. 97 + 131 + 165 + …, find S_{37}

$131 - 97 = 34$
$165 - 131 = 34$

Arithmetic

$t_n = 63 + 34n$

$$S_{37} = \sum_{n=1}^{n=37} 63 + 34n$$

$$\boxed{S_{37} = 26{,}233}$$

2. 1000 + 900 + 810 + …, find S_{22}

$\frac{900}{1000} = .9$

$\frac{810}{900} = .9$

Geometric

$t_n = 1111.\overline{1} * .9^n$ OR $t_n = 1000 * .9^{n-1}$

$$S_{22} = \sum_{n=1}^{n=22} 1000 * .9^{n-1}$$

* used because no rounding issues

$$\boxed{S_{22} = 9015.23}$$

The n-th partial sum of an ***arithmetic*** series is	$S_n = \frac{n}{2}(t_1 + t_n)$
The n-th partial sum of a ***geometric*** series is	$S_n = t_1 \cdot \frac{1-r^n}{1-r}$ r is common ratio

The series is either arithmetic or geometric. Find *n* for the given partial sum.

3. 97 + 101 + 105 + …, find n if S_n= 21,663

$101 - 97 = 4$

$105 - 101 = 4$

Arithmetic

$S_n = \frac{n}{2}(t_1 + t_n)$

$t_n = 97 + 4(n-1)$
$97 + 4n - 4$
$93 + 4n$

$21,663 = \frac{n}{2}(97 + t_n)$

$21,663 = \frac{n}{2}(97 + 93 + 4n)$

$21,663 = \frac{n}{2}(190 + 4n)$

$43,326 = n(190 + 4n)$

$43,326 = 190n + 4n^2$

$4n^2 + 190n - 43,326 = 0$

$n = 83$ or ~~$n = -130.5$~~

4. 13 + 26 + 52 + …, find n if S_n= 425,971

$\frac{26}{13} = 2$

$\frac{52}{26} = 2$

Geometric

$S_n = t_1 * \frac{1-r^n}{1-r}$

$425,971 = 13 * \frac{1-2^n}{1-2}$

$32767 = \frac{1-2^n}{-1}$

$-32767 = 1 - 2^n$

$-32768 = -2^n$

$32768 = 2^n$ → $\frac{\log 32768}{\log 2} = n$

$n = 15$