15.2 (con't)

**Notes 15.2** Sums and Products of Zeros

1. $f(x) = 5x^3 - 33x^2 + 58x - 24$
Find the zeros. .6, 2, 4

Factor out the leading coefficient 5 from the equation.

$f(x) = 5\left(x^3 - \frac{33}{5}x^2 + \frac{58}{5}x - \frac{24}{5}\right)$ OR $f(x) = 5(x^3 - 6.6x^2 + 11.6x - 4.8)$

$z_1 + z_2 + z_3 =$ $.6 + 2 + 4 = 6.6 \Rightarrow$ opp. of quad. coefficient

$z_1z_2z_3 =$ $(.6)(2)(4) = 4.8 \Rightarrow$ opp of constant

$z_1z_2 + z_1z_3 + z_2z_3 =$ $(.6)(2) + (.6)(4) + (2)(4) = 1.2 + 2.4 + 8 = 11.6 \Rightarrow$ linear coeff.

If $p(x) = ax^3 + bx^2 + cx + d$ has zeros $z_1$, $z_2$, *and* $z_3$ then

| | |
|---|---|
| $z_1 + z_2 + z_3 = -\frac{b}{a}$ | sum of the zeros |
| $z_1z_2z_3 = -\frac{d}{a}$ | product of the zeros |
| $z_1z_2 + z_1z_3 + z_2z_3 = \frac{c}{a}$ | sum of the pairwise products of the zeros |

2. In a cubic function, if the three zeros are x = -1.5, x = 1 and x = 5, write an equation where the leading coefficient is 1. $\rightarrow a = 1$

Sum: $-1.5 + 1 + 5 = 4.5 = -\frac{b}{a} \Rightarrow 4.5 = \frac{-b}{1}$ $b = -4.5$

Product: $(-1.5)(1)(5) = -7.5 = -\frac{d}{a} \Rightarrow -7.5 = \frac{-d}{1}$ $d = 7.5$

PWP: $(-1.5)(1) + (-1.5)(5) + (1)(5) = -1.5 + -7.5 + 5 = -4 = \frac{c}{a} = -4 = \frac{c}{1}$ $c = -4$

$\therefore f(x) = x^3 - 4.5x^2 - 4x + 7.5$

Find a particular equation of the cubic function, if the leading coefficient equals 1.

3. Sum of zeros = 9
   Product of zeros = 24
   Sum of pairwise products = 26

$f(x) = ax^3 + bx^2 + cx + d$

$9 = \frac{-b}{a} = \frac{-b}{1}$ $b = -9$

$24 = \frac{-d}{a} = \frac{-d}{1}$ $d = -24$

$26 = \frac{c}{a} = \frac{c}{1}$ $c = 26$

$f(x) = x^3 - 9x^2 + 26x - 24$

4. If two of the zeros of a cubic function are x =1 and x = -3 + *i* , write an equation where the leading coefficient is 1.

$x = 1$

$x = -3 + i$

$x = -3 - i$

$f(x) = x^3 + 5x^2 + 4x - 10$

Sum: $1 + (-3+i) + (-3-i)$

$1 + (-6) = -5$

$-5 = \frac{-b}{a} = \frac{-b}{1}$ $b = 5$

product: $(1)(-3+i)(-3-i)$

$(1)(9 + 3i - 3i - i^2)$

$(1)(9 - (-1)) = 10$

$10 = \frac{-d}{a} = \frac{-d}{1}$ $d = -10$

pwp: $1(-3+i) + 1(-3-i) + (-3+i)(-3-i)$

$(-3+i) + (-3-i) + 10$

$-6 + 10 = 4$

$4 = \frac{c}{a} = \frac{c}{1}$ $c = 4$