Notes 15.4 Rational Functions: Discontinuities and Limits

$f(x) = \frac{x^2-x-12}{x-4}$

4 | 1 -1 -12
↓ 4 12
1 3 0

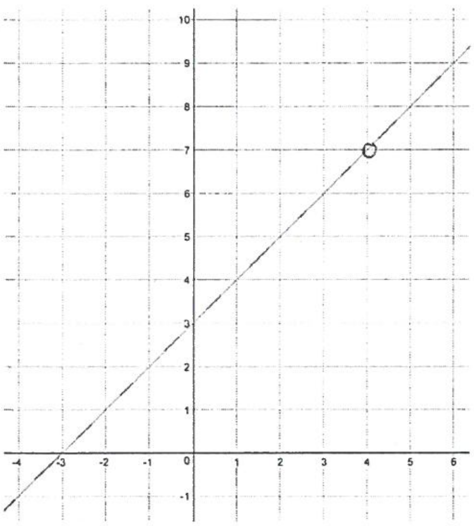
4 is a root of numerator

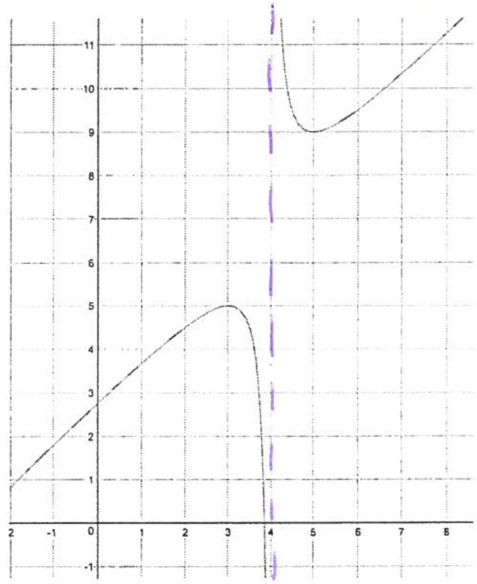
$g(x) = \frac{x^2-x-11}{x-4}$

4 | 1 -1 -11
↓ 4 12
1 3 1

remainder means 4 is NOT a root

Do the graphs of these similar equations look alike? They are each undefined at x = 4 or have a **discontinuity**.





Since the factor (x – 4) can be removed, it is called a **removable discontinuity**.

(if removed, have a continuous function)

no factors can be removed from g(x) so there is a **vertical asymptote**

As x approaches 4, f(x) approaches 7.

g(4) = undefined

$\lim_{x\to 4} f(x) =$ 7 This # is called the **limit.**

The **limit** $\lim_{x\to 4} g(x) = \infty$

to find limit

$f(x) = \frac{(x-4)(x+3)}{(x-4)}$ ① factor

$f(x) = (x+3)$ ② cancel

$f(x) = (4+3)$ ③ plug x=4 (restriction) into what's left

$f(x) = 7$

* See table on calc
Set Δ table to .01
table start to 3.9

$f(x) = \frac{x^3-5x^2+8x-6}{x-3}$ $g(x) = \frac{x^3-5x^2+8x-5}{x-3}$

Where is the **discontinuity**? at $x = 3$

Find the **limit** of each function at the discontinuity.

① find out if denom. is a factor of the numerator.

f(x)

$$\begin{array}{c|cccc} 3 & 1 & -5 & 8 & -6 \\ & \downarrow & 3 & -6 & 6 \\ \hline & 1 & -2 & 2 & \boxed{0} \end{array}$$

Since remainder is 0,
X-3 is factor of numerator;
so, this is a removable discontinuity

g(x)

$$\begin{array}{c|cccc} 3 & 1 & -5 & 8 & -5 \\ & \downarrow & 3 & -6 & 6 \\ \hline & 1 & -2 & 2 & \boxed{1} \end{array}$$

Since remainder is 1,
X-3 is NOT a factor of the numerator, so
this is a non-removable discontinuity

$$\boxed{\lim_{x \to 3} g(x) = \infty}$$

② to find limit, plug restriction (x value that makes denom = 0) into "answer" from the division.

"answer" → $x^2 - 2x + 2$

restriction → 3

$$\lim_{x \to 3} f(x) = (3)^2 - 2(3) + 2$$

$$\boxed{\lim_{x \to 3} f(x) = 5}$$