

Notes 15.4 Partial Fractions

1. $g(x) = \frac{4x-2}{x^2-x-6}$ where are discontinuities? $\frac{4x-2}{(x-3)(x+2)} \Rightarrow \boxed{@ x=3 \text{ ; } x=-2}$

a) List any real zeros of the function. $0 = \frac{4x-2}{x^2-x-6}$ $0 = 4x-2$ $2 = 4x$

$\boxed{x = \frac{1}{2}}$

b) Why are both discontinuities at $x = 3$ and $x = -2$ vertical asymptotes?

not factors of numerator

Add together $\frac{(x+2)2}{(x+2)x-3} + \frac{2(x-3)}{x+2(x-3)} \Rightarrow \frac{2(x+2) + 2(x-3)}{(x+2)(x-3)} = \frac{2x+4+2x-6}{(x+2)(x-3)} = \boxed{\frac{4x-2}{(x+2)(x-3)}}$

These are called **partial fractions**. We can reverse this adding process and break the rational function into partial fractions using the discontinuities.

Substitute $x = 3$ and cover up the $(x - 3)$ factor.

$\frac{4x-2}{(x+2)(x-3)}$ * for $x=3$

$\frac{4x-2}{x+2} \Rightarrow$ sub 3 in for all x 's except the one that will make it 0

$\frac{4(3)-2}{3+2}$

$\frac{12-2}{5} = \frac{10}{5}$

2 \Rightarrow this becomes the numerator for the $(x-3)$ We didn't use

$\frac{2}{x-3}$

Substitute $x = -2$ and cover up the $(x + 2)$.

$\frac{4x-2}{(x+2)(x-3)}$ * for $x=-2$

$\frac{4x-2}{x-3}$ sub in -2 for x

$\frac{4(-2)-2}{-2-3} = \frac{-10}{-5}$

$-2 \Rightarrow$ num. for $x+2$

$\frac{2}{x+2}$

$\frac{2}{x-3} + \frac{2}{x+2}$

$$2. \quad g(x) = \frac{7x-2}{x^2-x-2} \quad \frac{7x-2}{(x-2)(x+1)}$$

a) List any real zeros of the function.

$$0 = 7x - 2 \quad 2 = 7x$$

$$\boxed{x = \frac{2}{7}}$$

b) Find any discontinuities and identify what type they are.

$$\begin{aligned} @ x = 2 \\ @ x = -1 \end{aligned}$$

both non-removable

c) Resolve the function into partial fractions.

$$\frac{7x-2}{(x-2)(x+1)} \quad x=2$$

$$\frac{7x-2}{(x-2)(x+1)} \quad x=-1$$

$$\frac{7(2)-2}{2+1} = 4$$

$$\frac{7(-1)-2}{-1-2} = 3$$

$$\boxed{\frac{4}{x-2} + \frac{3}{x+1}}$$

$$3. \quad f(x) = \frac{3x-37}{(x+1)(x-4)}$$

a) List any real zeros of the function.

$$0 = 3x - 37 \quad 3x = 37$$

$$\boxed{x = \frac{37}{3} \approx 12.\bar{3}}$$

b) Find any discontinuities and identify what type they are.

$$\begin{aligned} @ x = -1 \\ @ x = 4 \end{aligned}$$

both non-removable

c) Resolve the function into partial fractions.

$$x=-1 \quad \frac{3(-1)-37}{-1-4} = \frac{-40}{-5} = 8$$

$$\boxed{\frac{8}{x+1} + \frac{-5}{x-4}}$$

$$x=4 \quad \frac{3(4)-37}{4+1} = \frac{-25}{5} = -5$$