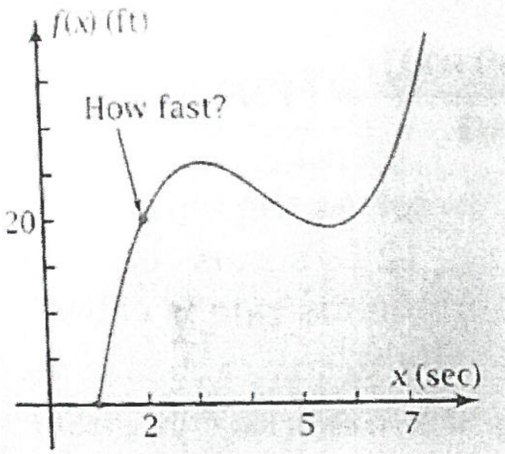
Key

Notes 15.5 Instantaneous Rate of Change: The Derivative

1. Suppose that a bird takes off from the ground at time x= 1 second. It climbs for a while, then dives for a while, and then swoops back up again. The figure shows what its height might be as a function of time.



From the graph you can tell that the bird still climbing at x = 2 sec. The question is, "At what *rate* is the bird climbing at the instant x = 2?"

You will learn how to calculate the **derivative** of certain kinds of functions, which tells you the **instantaneous rate.**

rate = slope = $\frac{\Delta y}{\Delta x}$

Suppose the bird's height is given by the function $f(x) = x^3 - 13x^2 + 52x - 40$

notation for derivative

$$\boxed{f'(x)} = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$$

Numerically: (use #'s)

* What's an instant? 1 sec? .1 sec? .01 sec?

* use 2 and 2.001

* $\frac{f(2.001) - f(2)}{2.001 - 2} = \frac{20.01199 - 20}{.001}$

$\boxed{\text{rate} = 11.99}$

* look @ table → $\lim_{x \to 2} f(x) = \boxed{12}$

Algebraically (use x)

$\frac{f(x) - f(2)}{x - 2} = \frac{(x^3 - 13x^2 + 52x - 40) - 20}{x - 2}$

$\frac{x^3 - 13x^2 + 52x - 60}{x - 2}$

$$\begin{array}{r|rrrr} 2 & 1 & -13 & 52 & -60 \\ & \downarrow & 2 & -22 & 60 \\ \hline & 1 & -11 & 30 & \boxed{0} \end{array}$$

find limit by putting 2 into result of synthetic division → $x^2 - 11x + 30$

$(2)^2 - 11(2) + 30$

$\boxed{12}$

The **instantaneous rate of change** of f(x) at x = c is called the **derivative** and is denoted $f'(x)$, which is pronounced "f prime of x". It is equal to the **limit** of the average rate as x approaches c.

$$f'(x) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

2. $f(x) = 2x^3 - 6x^2 + 2x + 4$

Find the instantaneous rate of change at x = 2 f(2) = 0

$$f'(x) = \lim_{x\to 2}\frac{f(x)-f(2)}{x-2} = \frac{(2x^3-6x^2+2x+4)-0}{x-2}$$

$$\frac{2x^3-6x^2+2x+4}{x-2}$$

$$\begin{array}{r|rrrr} 2 & 2 & -6 & 2 & 4 \\ & \downarrow & 4 & -4 & -4 \\ \hline & 2 & -2 & -2 & \boxed{0} \end{array}$$

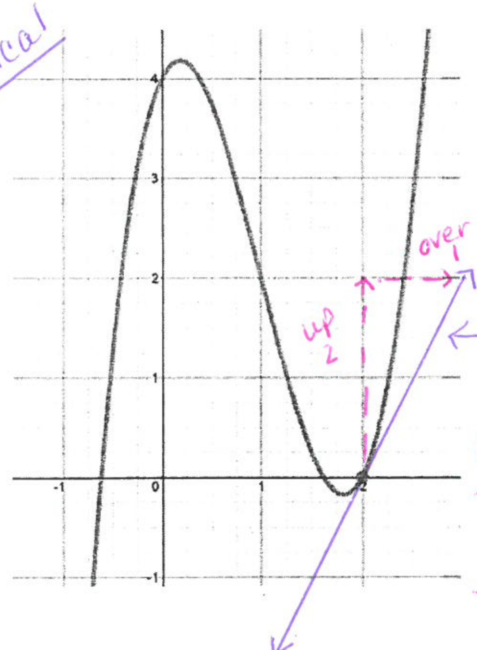
$2x^2-2x-2$ put in $x=2$

$2(2)^2-2(2)-2$

$\boxed{2}$

The value of the derivative of f(x) at x = c equals the **slope of the tangent line** to the graph of f at x = c.

Graphical



tangent line

Slope of tangent line is 2 --- this is the derivative

Find the equation of the tangent line at x = 2.

$y = mx + b$

$f(2)$ $f'(2) \rightarrow$ derivative @ $x=2$

$0 = 2(2) + b$

$0 = 4 + b$ $b = -4$

$\boxed{y = 2x - 4}$

*if line drawn (at left) were extended, it would cross the y-axis @ -4

Extra example: Find the instantaneous rate of change at x = 1.

$$\frac{f(x)-f(1)}{x-1} = \frac{(2x^3-6x^2+2x+4)-2}{x-1}$$

$$\frac{2x^3-6x^2+2x+2}{x-1} \rightarrow \begin{array}{r|rrrr} 1 & 2 & -6 & 2 & 2 \\ & \downarrow & 2 & -4 & -2 \\ \hline & 2 & -4 & -2 & \boxed{0} \end{array}$$

$2x^2-4x-2$ (plug in 1) $\rightarrow \boxed{-4}$