

Notes 2.2 Integration by Substitution

Use the suggested substitution to find the following indefinite integrals.

1. $\int (x-5)^3 dx$ $u = x-5$

$\int u^3 dx$ $\frac{du}{dx} = 1$
 $\int u^3 \cdot du$ $du = dx$

* must replace dx w/ du

$\frac{u^4}{4} + C$ $\frac{(x-5)^4}{4} + C$

2. $\int (3x+1)^4 dx$ $u = 3x+1$

$\frac{1}{3} \int u^4 du$ $\frac{du}{dx} = 3$
 $\frac{1}{3} \frac{u^5}{5}$ $du = 3 dx$
 $\frac{u^5}{15}$ $\frac{1}{3} du = dx$

$\frac{(3x+1)^5}{15} + C$

3. $\int \frac{1}{(2x+3)^2} dx$ $u = 2x+3$

$\frac{1}{2} \int u^{-2} du$ $\frac{du}{dx} = 2$
 $\frac{1}{2} \frac{u^{-1}}{-1}$ $du = 2 dx$
 $\frac{u^{-1}}{-2}$ $\frac{1}{2} du = dx$

$\frac{(2x+3)^{-1}}{-2} + C$

4. $\int \frac{1}{(7-x)} dx$ $u = 7-x$

$-1 \int u^{-1} du$ $\frac{du}{dx} = -1$
 $-1 \ln|u|$ $du = -dx$
 $-du = dx$

$-1 \ln|7-x| + C$
 or $\ln|7-x|^{-1} + C$

5. $\int x(x^2+9)^5 dx$ $u = x^2+9$

$\int u^5 x dx$ $\frac{du}{dx} = 2x$
 $\frac{1}{2} \int u^5 du$ $du = 2x dx$
 $\frac{1}{2} \frac{u^6}{6}$ $\frac{1}{2} du = x dx$

$\frac{u^6}{12}$ $\frac{(x^2+9)^6}{12} + C$

6. $\int x\sqrt{4-x^2} dx$ $u = 4-x^2$

$\int u^{\frac{1}{2}} x dx$ $\frac{du}{dx} = -2x$
 $-\frac{1}{2} \int u^{\frac{1}{2}} du$ $du = -2x dx$
 $-\frac{1}{2} \frac{u^{\frac{3}{2}}}{\frac{3}{2}}$ $-\frac{1}{2} du = x dx$

$-\frac{1}{2} \cdot \frac{2}{3} \cdot (4-x^2)^{\frac{3}{2}}$
 $-\frac{1}{3} (4-x^2)^{\frac{3}{2}} + C$