5-2 Verifying Trigonometric Identities

Verify each identity.

1. $(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$

SOLUTION:

$(\sec^2\theta-1)\cos^2\theta$

$=(\tan^2\theta)\cos^2\theta$ Pythagorean Identity

$=\left(\frac{\sin^2\theta}{\cos^2\theta}\right)\cos^2\theta$ Quotient Identity

$=\sin^2\theta$ Multiply and divide out common factor.

2. $\sec^2\theta\,(1-\cos^2\theta)=\tan^2\theta$

SOLUTION:

$\sec^2\theta(1-\cos^2\theta)$

$=\sec^2\theta-\sec^2\theta\cos^2\theta$ Distributive Property

$=\sec^2\theta-\frac{1}{\cos^2\theta}\cdot\cos^2\theta$ Reciprocal Identity

$=\sec^2\theta-1$ Multiply and divide out common factor.

$=\tan^2\theta$ Pythagorean Identity

3. $\sin\theta-\sin\theta\cos^2\theta=\sin^3\theta$

SOLUTION:

$\sin\theta-\sin\theta\cos^2\theta$

$=\sin\theta(1-\cos^2\theta)$ Factor.

$=\sin\theta\sin^2\theta$ Pythagorean Identity

$=\sin^3\theta$ Multiply.

4. $\csc\theta-\cos\theta\cot\theta=\sin\theta$

SOLUTION:

$\csc\theta-\cos\theta\cot\theta$

$=\frac{1}{\sin\theta}-\cos\theta\left(\frac{\cos\theta}{\sin\theta}\right)$ Reciprocal/Quotient Identities

$=\frac{1-\cos^2\theta}{\sin\theta}$ Write with a com denom

$=\frac{\sin^2\theta}{\sin\theta}$ Pythagorean Identity

$=\sin\theta$ Divide out factor of $\sin\theta$

5. $\cot^2\theta\csc^2\theta-\cot^2\theta=\cot^4\theta$

SOLUTION:

$\cot^2\theta\csc^2\theta-\cot^2\theta$

$=\cot^2\theta(\csc^2\theta-1)$ Factor.

$=\cot^2\theta\cot^2\theta$ Pythagorean Identity

$=\cot^4\theta$ Multiply and add exponents.

6. $\tan\theta\csc^2\theta-\tan\theta=\cot\theta$

SOLUTION:

$\tan\theta\csc^2\theta-\tan\theta$

$=\tan\theta(\csc^2\theta-1)$ Factor

$=\tan\theta\cot^2\theta$ Pythagorean Identity

$=\frac{\sin\theta}{\cos\theta}\frac{\cos^2\theta}{\sin^2\theta}$ Quotient Identities

$=\frac{\cos\theta}{\sin\theta}$ Multiply and divide common factors.

$=\cot\theta$ Quotient Identity

7. $\frac{\sec\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}=\cot\theta$

SOLUTION:

$\frac{\sec\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}$

$=\frac{\frac{1}{\cos\theta}}{\sin\theta}-\frac{\sin\theta}{\cos\theta}$ Reciprocal Identity

$=\frac{1}{\sin\theta\cos\theta}-\frac{\sin^2\theta}{\sin\theta\cos\theta}$ Common denominator

$=\frac{1-\sin^2\theta}{\sin\theta\cos\theta}$ Write with a com denom

$=\frac{\cos^2\theta}{\sin\theta\cos\theta}$ Pythagoren Identity

$=\frac{\cos\theta}{\sin\theta}$ Divide out $\cos\theta$

$=\cot\theta$ Quotient Identity

8. $\frac{\sin\theta}{1-\cos\theta}+\frac{1-\cos\theta}{\sin\theta}=2\csc\theta$

SOLUTION:

$\frac{\sin\theta}{1-\cos\theta}+\frac{1-\cos\theta}{\sin\theta}$

$=\frac{\sin\theta}{\sin\theta}\cdot\frac{\sin\theta}{1-\cos\theta}+\frac{1-\cos\theta}{1-\cos\theta}\cdot\frac{1-\cos\theta}{\sin\theta}$ Rewrite using com denom

$=\frac{\sin^2\theta}{\sin\theta(1-\cos\theta)}+\frac{1-2\cos\theta+\cos^2\theta}{\sin\theta(1-\cos\theta)}$ Mulitply

$=\frac{\sin^2\theta+\cos^2\theta+1-2\cos\theta}{\sin\theta(1-\cos\theta)}$ Write with a com denom

$=\frac{1+1-2\cos\theta}{\sin\theta(1-\cos\theta)}$ Pythagorean Identity

$=\frac{2-2\cos\theta}{\sin\theta(1-\cos\theta)}$ Add

$=\frac{2(1-\cos\theta)}{\sin\theta(1-\cos\theta)}$ Factor

$=\frac{2}{\sin\theta}$ Divide out $(1-\cos\theta)$

$=2\csc\theta$ Reciprocal Identity

5-2 Verifying Trigonometric Identities

9. $\dfrac{\cos\theta}{1+\sin\theta}+\tan\theta=\sec\theta$

SOLUTION:

$\dfrac{\cos\theta}{1+\sin\theta}+\tan\theta$

$=\dfrac{\cos\theta}{1+\sin\theta}+\dfrac{\sin\theta}{\cos\theta}$ Quotient Identity

$=\dfrac{\cos\theta}{\cos\theta}\cdot\dfrac{\cos\theta}{1+\sin\theta}+\dfrac{1+\sin\theta}{1+\sin\theta}\cdot\dfrac{\sin\theta}{\cos\theta}$ Rewrite 1 with com denom

$=\dfrac{\cos^2\theta}{\cos\theta(1+\sin\theta)}+\dfrac{\sin\theta+\sin^2\theta}{(1+\sin\theta)\cos\theta}$ Multiply

$=\dfrac{\cos^2\theta+\sin\theta+\sin^2\theta}{\cos\theta(1+\sin\theta)}$ Write as a single fraction

$=\dfrac{1+\sin\theta}{\cos\theta(1+\sin\theta)}$ Pythagorean Identity

$=\dfrac{1}{\cos\theta}$ Divide out $(1+\sin\theta)$

$=\sec\theta$ Reciprocal Identity

10. $\dfrac{\sin\theta}{1-\cot\theta}+\dfrac{\cos\theta}{1-\tan\theta}=\sin\theta+\cos\theta$

SOLUTION:

$\dfrac{\sin\theta}{1-\cot\theta}+\dfrac{\cos\theta}{1-\tan\theta}$

$=\dfrac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}+\dfrac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}$ Quotient Identity

$=\dfrac{\sin\theta}{\frac{\sin\theta}{\sin\theta}-\frac{\cos\theta}{\sin\theta}}+\dfrac{\cos\theta}{\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}$ Rewrite using com denom

$=\dfrac{\sin\theta}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\dfrac{\cos\theta}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$ Write denom as single fractions

$=\dfrac{\sin^2\theta}{\sin\theta-\cos\theta}+\dfrac{\cos^2\theta}{\cos\theta-\sin\theta}$ Simplify fractions

$=\dfrac{\sin^2\theta}{\sin\theta-\cos\theta}-\dfrac{\cos^2\theta}{\sin\theta-\cos\theta}$ Factor out -1

$=\dfrac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}$ Write as a single fraction

$=\dfrac{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}{\sin\theta-\cos\theta}$ Factor numerator

$=\sin\theta+\cos\theta$ Divide out $(\sin\theta-\cos\theta)$

11. $\dfrac{1}{1-\tan^2\theta}+\dfrac{1}{1-\cot^2\theta}=1$

SOLUTION:

$\dfrac{1}{1-\tan^2\theta}+\dfrac{1}{1-\cot^2\theta}$

$=\dfrac{1}{1-\frac{\sin^2\theta}{\cos^2\theta}}+\dfrac{1}{1-\frac{\cos^2\theta}{\sin^2\theta}}$ Quotient Identity

$=\dfrac{1}{\frac{\cos^2\theta}{\cos^2\theta}-\frac{\sin^2\theta}{\cos^2\theta}}+\dfrac{1}{\frac{\sin^2\theta}{\sin^2\theta}-\frac{\cos^2\theta}{\sin^2\theta}}$ Rewrite 1 using com denom

$=\dfrac{1}{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}+\dfrac{1}{\frac{\sin^2\theta-\cos^2\theta}{\sin^2\theta}}$ Write denom as single fractions

$=\dfrac{\cos^2\theta}{\cos^2\theta-\sin^2\theta}+\dfrac{\sin^2\theta}{\sin^2\theta-\cos^2\theta}$ Simplify fractions

$=\dfrac{\cos^2\theta}{\cos^2\theta-\sin^2\theta}+\dfrac{-\sin^2\theta}{\cos^2\theta-\sin^2\theta}$ Common denominator

$=\dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta-\sin^2\theta}$ Write as a single fraction

$=1$ Divide out $(\cos^2\theta-\sin^2\theta)$

12. $\dfrac{1}{\csc\theta+1}+\dfrac{1}{\csc\theta-1}=2\sec^2\theta\sin\theta$

SOLUTION:

$\dfrac{1}{\csc\theta+1}+\dfrac{1}{\csc\theta-1}$

$=\dfrac{\csc\theta-1}{\csc\theta-1}\cdot\dfrac{1}{\csc\theta+1}+\dfrac{\csc\theta+1}{\csc\theta+1}\cdot\dfrac{1}{\csc\theta-1}$ Common denominator

$=\dfrac{\csc\theta-1}{\csc^2\theta-1}+\dfrac{\csc\theta+1}{\csc^2\theta-1}$ Multiply

$=\dfrac{2\csc\theta}{\csc^2\theta-1}$ Write as a single fraction

$=\dfrac{2\csc\theta}{\cot^2\theta}$ Pythagorean Identity

$=\dfrac{2\left(\frac{1}{\sin\theta}\right)}{\frac{\cos^2\theta}{\sin^2\theta}}$ Reciprocal/Quotient Identities

$=\dfrac{2}{\sin\theta}\cdot\dfrac{\sin^2\theta}{\cos^2\theta}$ Rewrite using multiplication

$=\dfrac{2\sin\theta}{\cos^2\theta}$ Multiply

$=\left(\dfrac{2}{\cos^2\theta}\right)\sin\theta$ Factor

$=2\sec^2\theta\sin\theta$ Reciprocal Identity

13. $(\csc\theta-\cot\theta)(\csc\theta+\cot\theta)=1$

SOLUTION:

$(\csc\theta-\cot\theta)(\csc\theta+\cot\theta)$

$=\csc^2\theta-\cot^2\theta$ Multiply.

$=1$ Pythagorean Identity

14. $\cos^4\theta-\sin^4\theta=\cos^2\theta-\sin^2\theta$

SOLUTION:

$\cos^4\theta-\sin^4\theta$

$=(\cos^2\theta+\sin^2\theta)(\cos^2\theta-\sin^2\theta)$ Factor

$=1(\cos^2\theta-\sin^2\theta)$ Pythagorean Identity

$=\cos^2\theta-\sin^2\theta$ Multiply

15. $\dfrac{1}{1-\sin\theta}+\dfrac{1}{1+\sin\theta}=2\sec^2\theta$

SOLUTION:

$\dfrac{1}{1-\sin\theta}+\dfrac{1}{1+\sin\theta}$

$=\dfrac{1+\sin\theta}{1+\sin\theta}\cdot\dfrac{1}{1-\sin\theta}+\dfrac{1-\sin\theta}{1-\sin\theta}\cdot\dfrac{1}{1+\sin\theta}$ Common denominator

$=\dfrac{1+\sin\theta}{1-\sin^2\theta}+\dfrac{1-\sin\theta}{1-\sin^2\theta}$ Multiply

$=\dfrac{2}{1-\sin^2\theta}$ Write as a single fraction

$=\dfrac{2}{\cos^2\theta}$ Pythagorean Identity

$=2\sec^2\theta$ Reciprocal Identity