

12-2 Writing Equations of a Hyperbola

Parametric Equations of Hyperbola

Opens in x-direction (left and right)

$$\begin{aligned} x &= \sec T + h \\ y &= \tan T + k \end{aligned}$$

Opens in y-direction (up and down)

$$\begin{aligned} x &= \tan T + h \\ y &= \sec T + k \end{aligned}$$

**opens in direction where sec T is

Write parametric equations. (same pictures from yesterday's notes to check)

1. $\left(\frac{x-1}{4}\right)^2 - \left(\frac{y+3}{2}\right)^2 = 1$

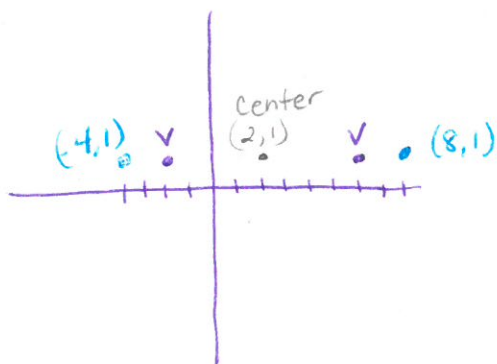
$$\begin{aligned} x &= 1 + 4 \sec T \\ y &= -3 + 2 \tan T \end{aligned}$$

2. $-\left(\frac{x+5}{2}\right)^2 + \left(\frac{y-4}{3}\right)^2 = 1$

$$\begin{aligned} x &= -5 + 2 \tan T \\ y &= 4 + 3 \sec T \end{aligned}$$

Write a Cartesian equation of the hyperbola that satisfies each set of conditions.

3. vertices (-2, 1) and (6, 1) and foci (-4, 1) and (8, 1)



$$e = \frac{c}{a}$$

$$e = \frac{6}{4} = \frac{3}{2}$$

Slopes of asymptotes: $\pm \frac{\sqrt{20}}{4}$

$$\text{OR } \pm \frac{\sqrt{5}}{2}$$

center: (2, 1)

$$a = 4$$

$$c = 6$$

$$c^2 = a^2 + b^2$$

$$36 = 16 + b^2$$

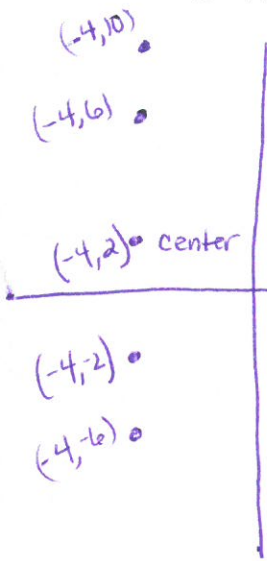
$$20 = b^2$$

$$\sqrt{20} = b$$

$$\boxed{\left(\frac{x-2}{4}\right)^2 - \left(\frac{y-1}{\sqrt{20}}\right)^2 = 1}$$

$$\boxed{\begin{aligned} x &= 2 + 4 \sec T \\ y &= 1 + \sqrt{20} \tan T \end{aligned}}$$

4. vertices $(-4, 6)$ and $(-4, -2)$ and foci $(-4, 10)$ and $(-4, -6)$



$$a = 4$$

$$c = 8$$

$$c^2 = a^2 + b^2$$

$$64 = 16 + b^2$$

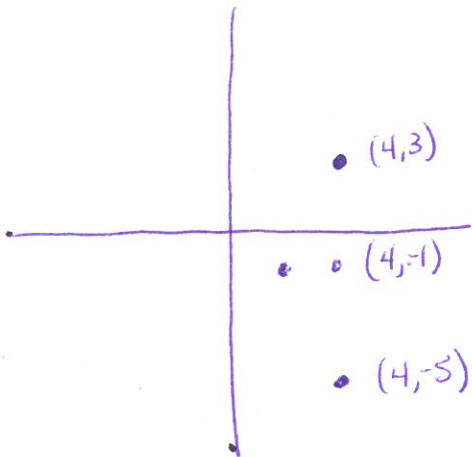
$$\sqrt{48} = b$$

$$-\left(\frac{x+4}{\sqrt{48}}\right)^2 + \left(\frac{y-2}{4}\right)^2 = 1$$

$$x = -4 + \sqrt{48} \tan T$$

$$y = 2 + 4 \sec T$$

5. vertices $(4, 3)$ and $(4, -5)$, conjugate axis length of 4



$$a = 4$$

$b = 2$ (axis is entire length, need radius)

$$-\left(\frac{x-4}{2}\right)^2 + \left(\frac{y+1}{4}\right)^2 = 1$$

$$x = 4 + 2 \tan T$$

$$y = -1 + 4 \sec T$$